



# TETHERED GRAVITY LABORATORIES STUDY

CONTRACT NAS 9-17877

QUARTERLY PROGRESS REPORT # 7

FOR THE PERIOD

25 AUGUST 1989 THROUGH 24 NOVEMBER 1989

DECEMBER 1989

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1.0 INTRODUCTION

This is the 7th quarterly progress report submitted by AERITALIA - GSS (AIT) under contract NAS9-17877 "Tethered Gravity Laboratories Study". This report covers the period from 25 AUG. 1989 through 24 NOV. 1989.

## 2.0 TECHNICAL STATUS

### 2.1 WORK ACCOMPLISHED DURING REPORTING PERIOD

On September 26/27/28 the Mid Term Review of the study was held at AERITALIA - Torino (Italy).

All the technical activities performed to date were presented and main results were discussed with reference to the Task 1, Task 2 (completed) and Task 3.

During the meeting a contractual modification was signed by AERITALIA, concerning the introduction of a new study area (Task 4 - Attitude Tether Stabilizer). That was a consequence of a partial redirection of the Task 1 agreed during the 1st Status Review of the Study (25 October 1988).

Due to schedule constraints, the initial AERITALIA proposal was revised in order to allow a four month schedule for this new task.

The technical approach for the "Attitude Tether Stabilizer" Task was also discussed during the meeting.

A new planning for the remaining activities of the study was implemented and it is reported in section 3.0.

During the reporting period the study task 3.0 (Variable Gravity Laboratory) was carried on, and the new Task 4.0 (Attitude Tether Stabilizer) was started.

Finally, as requested by NASA-JSC, the Task Report for the Task 1 (Active C.O.G. Control) was prepared and its final delivery is expected for December 1989.

#### 2.1.1 Variable Gravity Laboratory

a) An extensive analysis of the solar illumination of the VGL was performed, as requested by NASA-JSC, in order to determine the optimal accommodation of the body-mounted solar arrays and thermal radiators. The results of this analysis are reported in Appendix A.

b) The conceptual configuration of the VGL system is under investigation in terms of payload module technology, VGL technology and VGL system interfaces with the Space Station.

## 2.1.2 Attitude Tether Stabilizer

The major effort during reporting period was dedicated to this new task due to the tight schedule.

### 2.1.2.1 Configuration Analysis (AIT)

- a) The basic characteristics of the Space Station have been outlined from a minimum of available informations and relying on their logical consequences.
- b) A reference attitude control system was assumed in order to evaluate the tether stabilizer effectiveness.
- c) Station attitude control features were analysed with particular reference to the nominal TEA Flight Mode.
- d) A preliminary analysis of stability was performed with reference to a tether Space Station.
- e) Preliminary Results:
  - It appears possible to achieve stability around pitch and roll-yaw axes in a passive way by a proper tether system.
  - It was discovered the existence of a stability window for the tether stabilizer sizing.
- f) The attitude tether stabilizer properties were designed for the assumed Space Station Configuration.
- g) See Appendix B for detailed technical discussion.

### 2.1.2.2 Dynamics Analysis (SAO)

The control of the Space Station attitude by means of a tethered system has been analyzed. Specifically, the influence of a ballast tethered to the Station upon the Station attitude dynamics when no other attitude device is activated, has been numerically simulated.

The following simulation runs were performed:

- 1) SS attitude dynamics acted upon by tether and gravity gradient torques.
- 2) Same as 1) except for the aerodynamic torques and forces that were activated.

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3) Same as 2) except for the principal body frame that was rotated by a  $-6.27^\circ$  pitch angle.

See Appendix C for the complete technical reporting.

## 2.2 WORK PLANNED FOR THE NEXT REPORTING PERIOD

### 2.2.1 Variable Gravity Laboratory

The conceptual configuration of the VGL system will be completed in terms of payload module technology, VGL technology and VGL system interfaces with the Space Station.

### 2.2.2 Attitude Tether Stabilizer

The analysis of tether Stabilizer attitude dynamics will be further developed.  
The feasibility of using one or more tetherized masses will be investigated in order to coadiuvate the baseline Space Station ACS.

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### 3.0 SCHEDULE ASSESSMENT

As a result of the contractual modification described in para. 2.1, a new planning for the remaining activities of the study was implemented and it is outlined in the attached schedule.

During the reporting period the planned activities have been performed as follows:

#### Active C.o.G. Control

- Completed. Task Report delivered to NASA-JSC.

#### Low Gravity Processes Identification

- Completed. Task Report delivered to NASA-JSC.

#### Variable Gravity Laboratory

- Concept Design & Engineering Analysis sub-task is completed.
- Control Strategies sub-task is completed.
- Technology requirements sub-task is in progress, 70% completed.

#### Attitude Tether Stabilizer

- Configuration Analysis sub-task is in progress, 50% completed.
- Dynamics Analysis sub-task is in progress, 50% completed.





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#### 4.0 PROBLEMS AND CONCERNS

No problems encountered during reporting period.

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APPENDIX A  
VARIABLE GRAVITY LABORATORY

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## ANALYSIS OF THE SOLAR ILLUMINATION OF THE VGL

After a preliminary study of the power subsystem for the VGL (see Q.P.R. #4), batteries and solar arrays have been selected as power sources.

In order to avoid the generation of additional disturbances to the microgravity environment, it is foreseen to mount the solar cells directly on the elevator body, i.e. without making use of fixed or Sun-orientable solar panels wings.

An analysis of the variation of the solar illumination of the elevator surfaces during each orbit is therefore needed for determining the optimal accommodation of the solar arrays.

The elevator has been modelled as a parallelepiped orbiting the Earth at a mean altitude of 352 Km, with the +Z axis along the radial direction (downward), the +X axis along the flight direction, and the +Y axis pointing in the opposite direction of the orbit angular momentum vector (see fig A.1). The orbit inclination is  $28.5^\circ$ .

The solar illumination angle ( $\beta$ ) on each VGL side, defined as the angle between the direction of the incoming radiation and the side itself (fig A.2), is function of the longitude of the ascending node ( $\Omega$ ), the Sun coordinates, and the position of the elevator along the orbit.

The orbital mean value of  $\beta$  (outside the eclipse) for the +X, +Y, +Z sides, has been then computed for different orientations of the line of nodes of the elevator orbit, and for the Sun positions corresponding to each day of the year. Results are reported in the graphs A.3 - A.8 (0 values of  $\langle\beta\rangle$  refers to the case in which the corresponding VGL side is in shadow).

Among the means of the illumination angles, the highest values are reached by  $\langle\beta_{+y}\rangle$  (note also that  $\beta_{+y}$  is practically constant during each orbit as the sides +Y are parallel to the orbital plane). Nevertheless, solar panels placed on those sides are illuminated with acceptable angles of incidence only for a restricted set of combinations  $\Omega$ -Sun direction.

Surfaces +X and -Z, on the contrary, are illuminated with lower mean angles of incidence, but the sets of the conditions determining acceptable values of  $\langle\beta\rangle$  are wider (and almost superimposed for these three surfaces).

$\beta_{+z}$ , instead, never exceed values greater than few degrees as the the corresponding VGL side is kept Earth pointing through the mission (in addition the +Z side very probably will be used for accommodating the interface with the Space Station.)

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By taking into account the results of this analysis together with the other configuration requirement, it has been decided to utilise the sides  $\pm X$ ,  $-Z$ , and  $+Y$  for accommodating the solar cells. This solution allow to have one or more solar arrays illuminated with mean angles of incidence very close to maximum possible value for many combinations (day of the year)-(longitude of ascending node), thus allowing a wide launch window for the VGL missions.

In addition, for that launch window, the  $-Y$  surface is almost always completely in shadow, and therefore can be exploited for mounting the thermal control radiators and the star trackers required for the VGL attitude reconstruction.

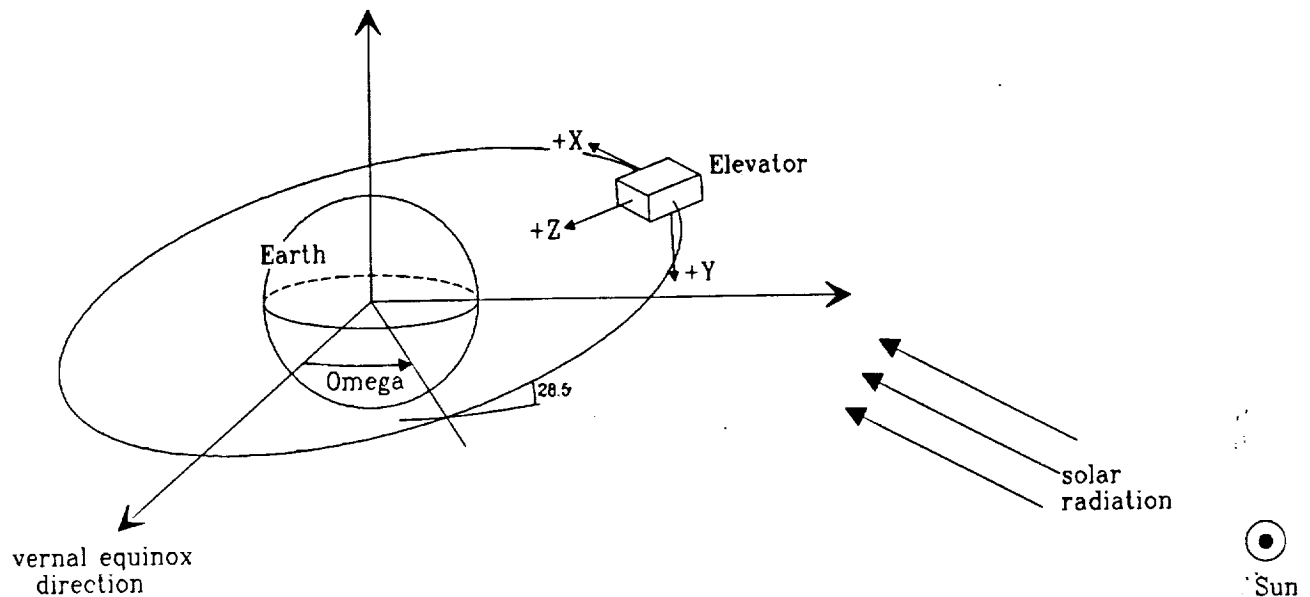


FIGURE A.1 VGL orbit and orientation.

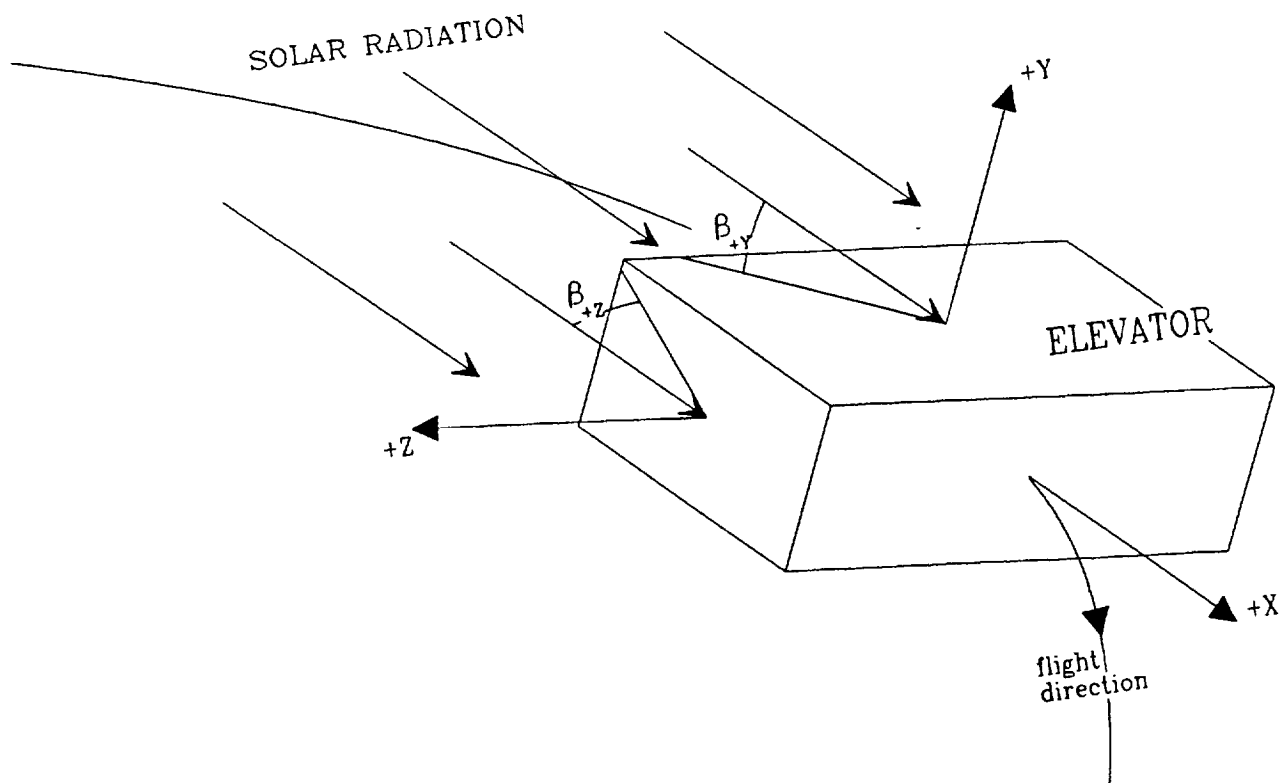


FIGURE A.2 Definition of the illumination angles.

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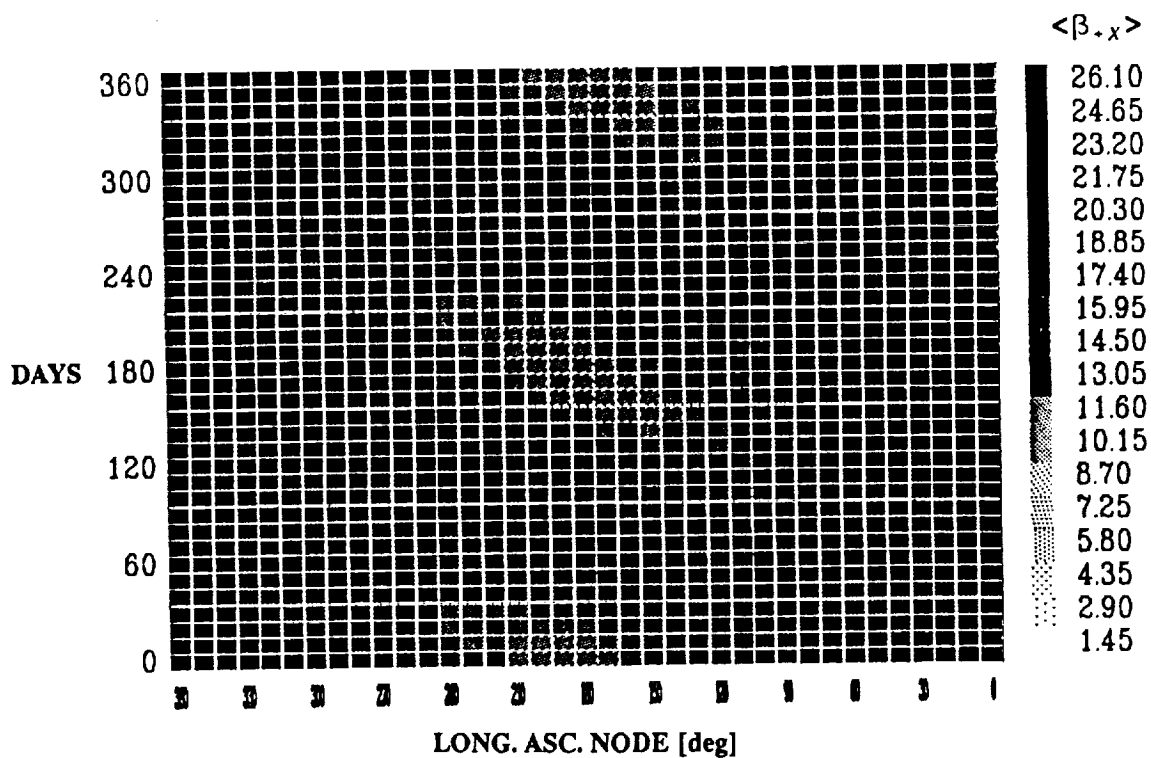
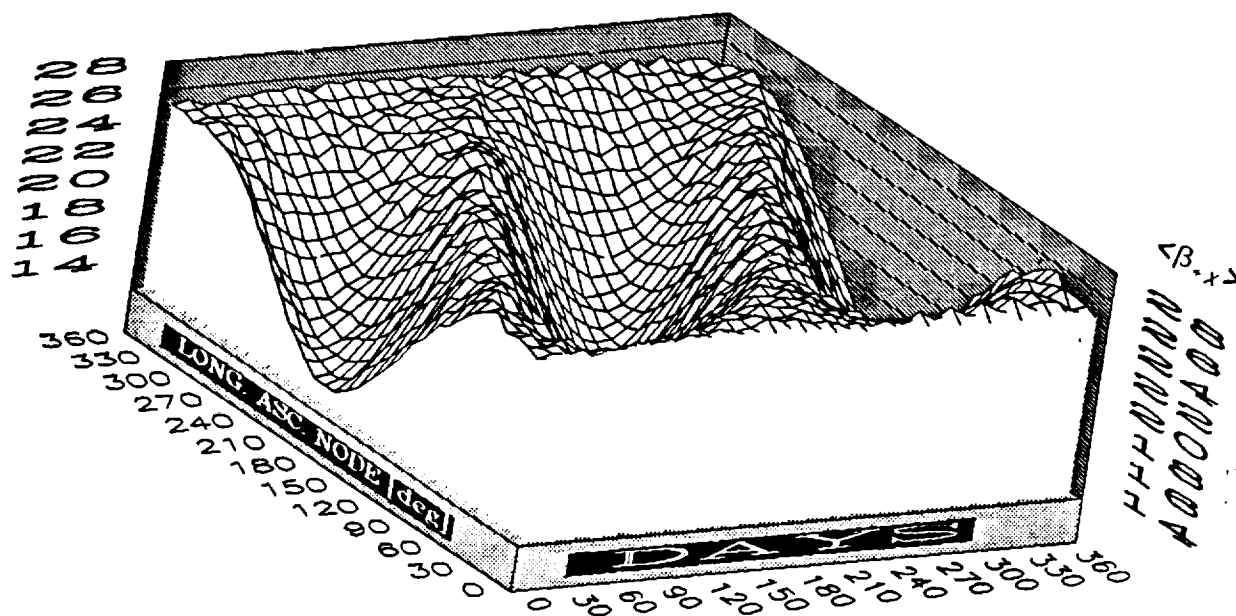


FIGURE A.3  $\langle \beta_{+X} \rangle$

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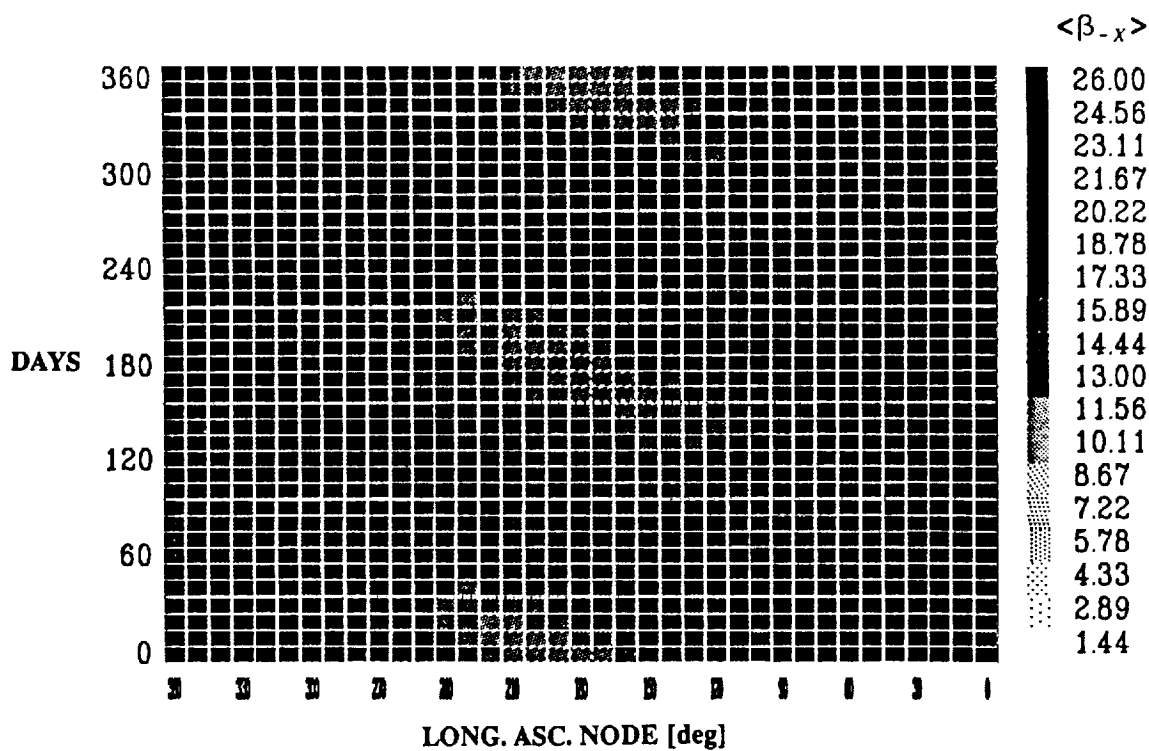
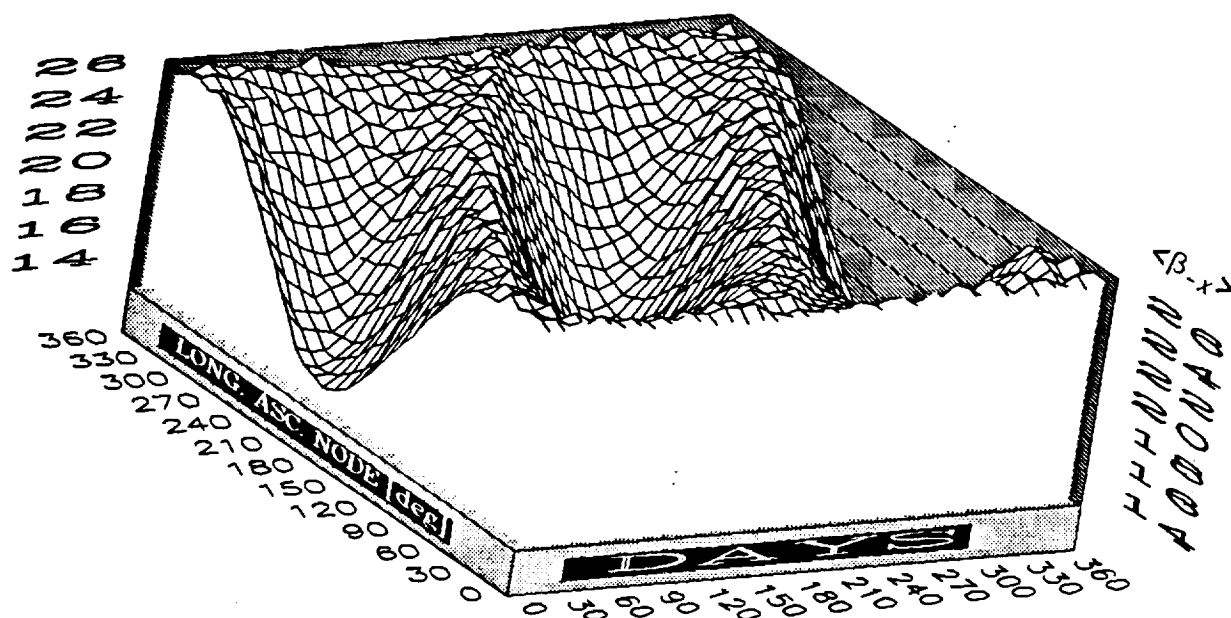


FIGURE A.4  $\langle \beta_x \rangle$



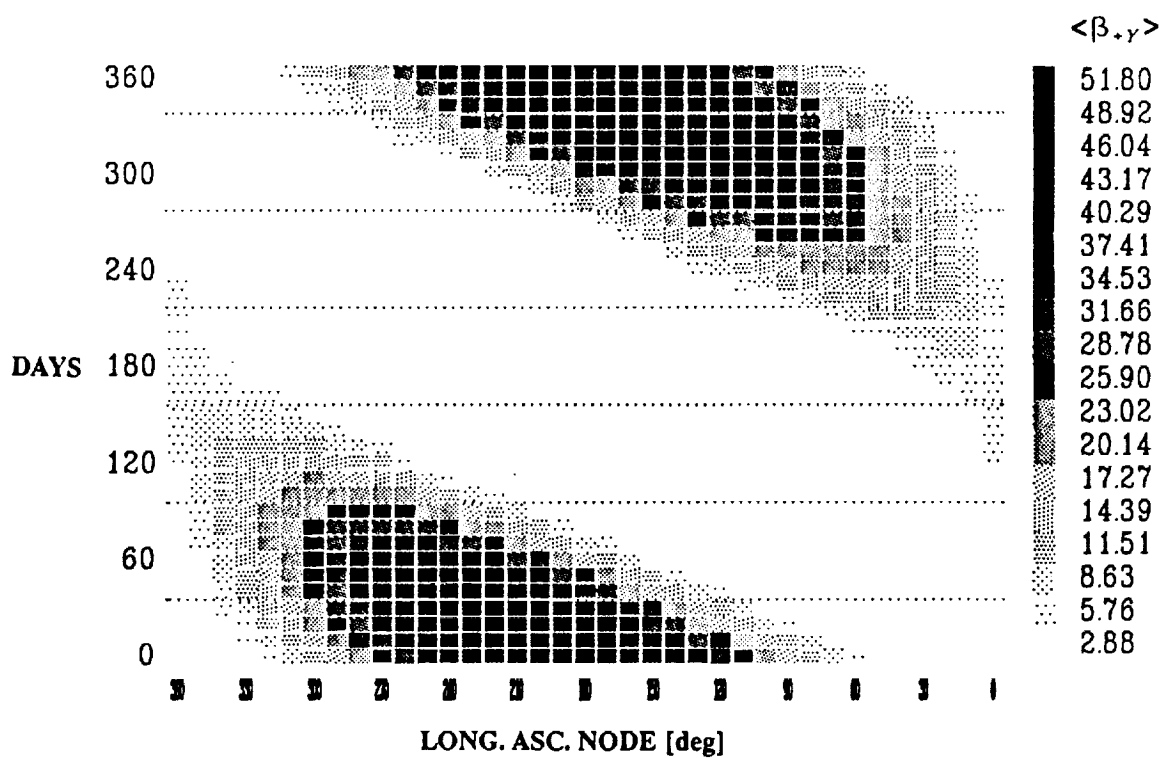
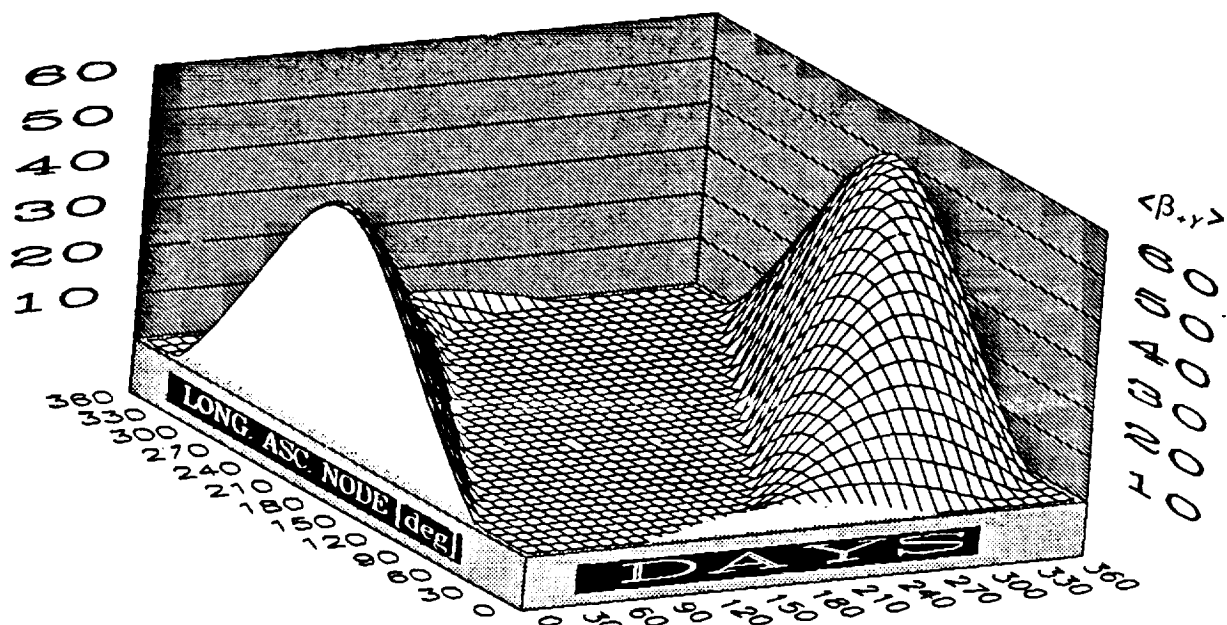


FIGURE A.5  $\langle \beta_y \rangle$

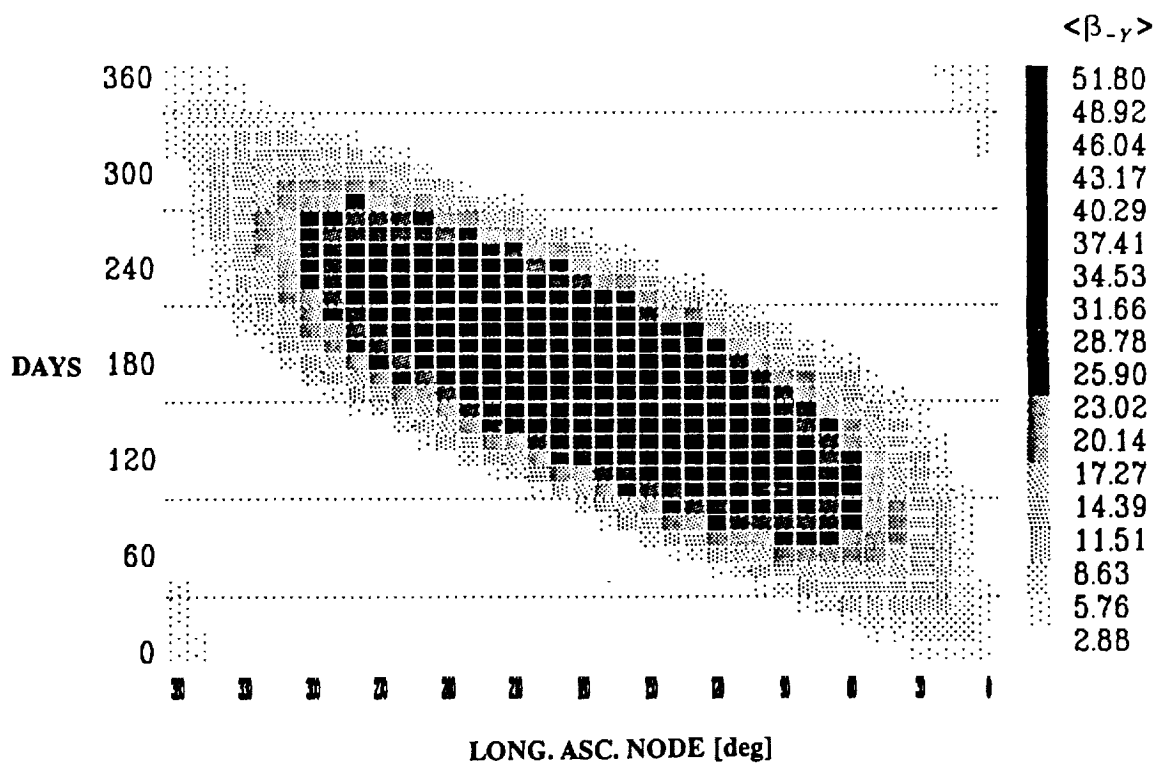
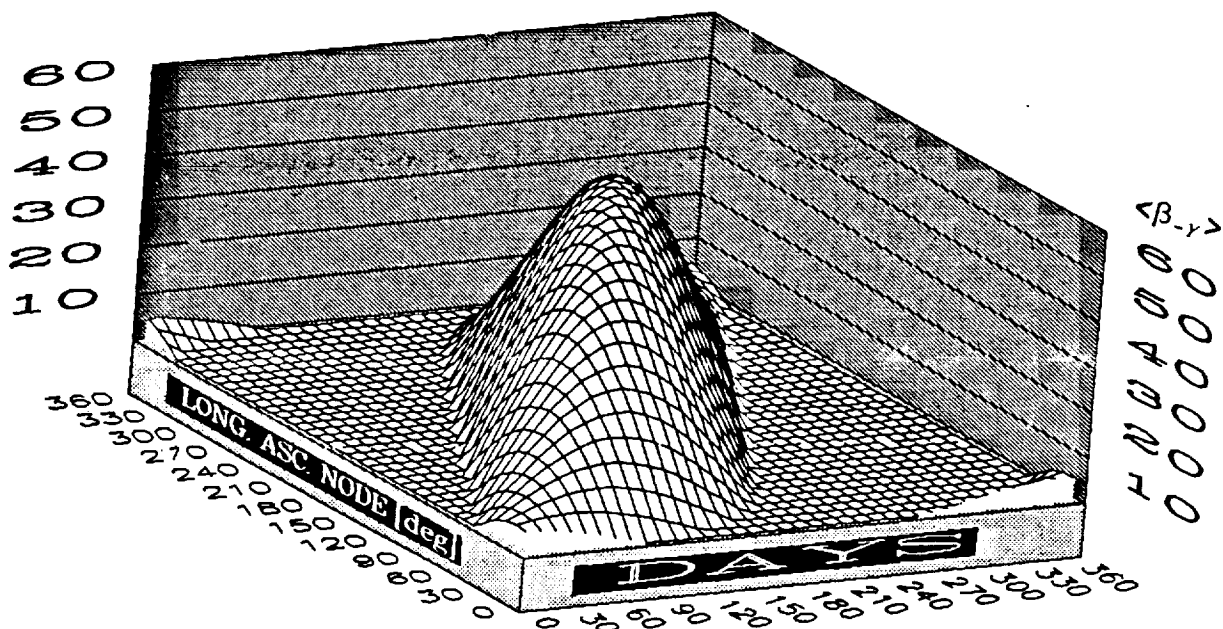


FIGURE A.6  $\langle \beta_y \rangle$

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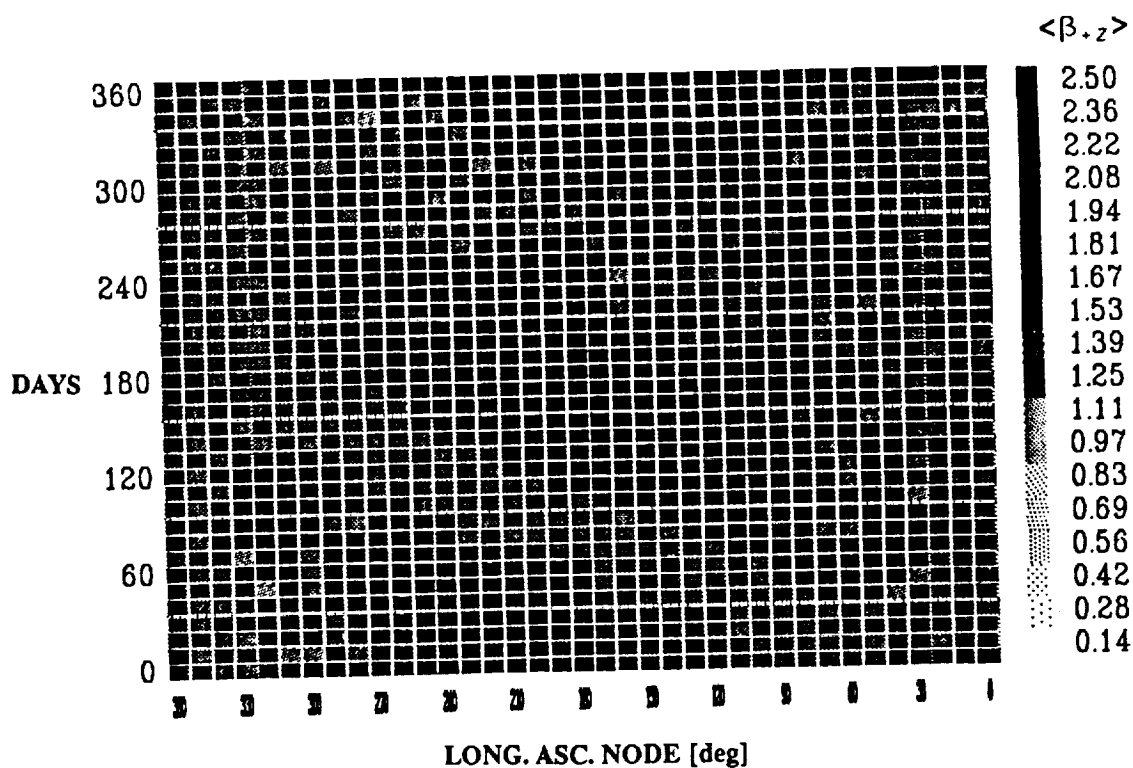
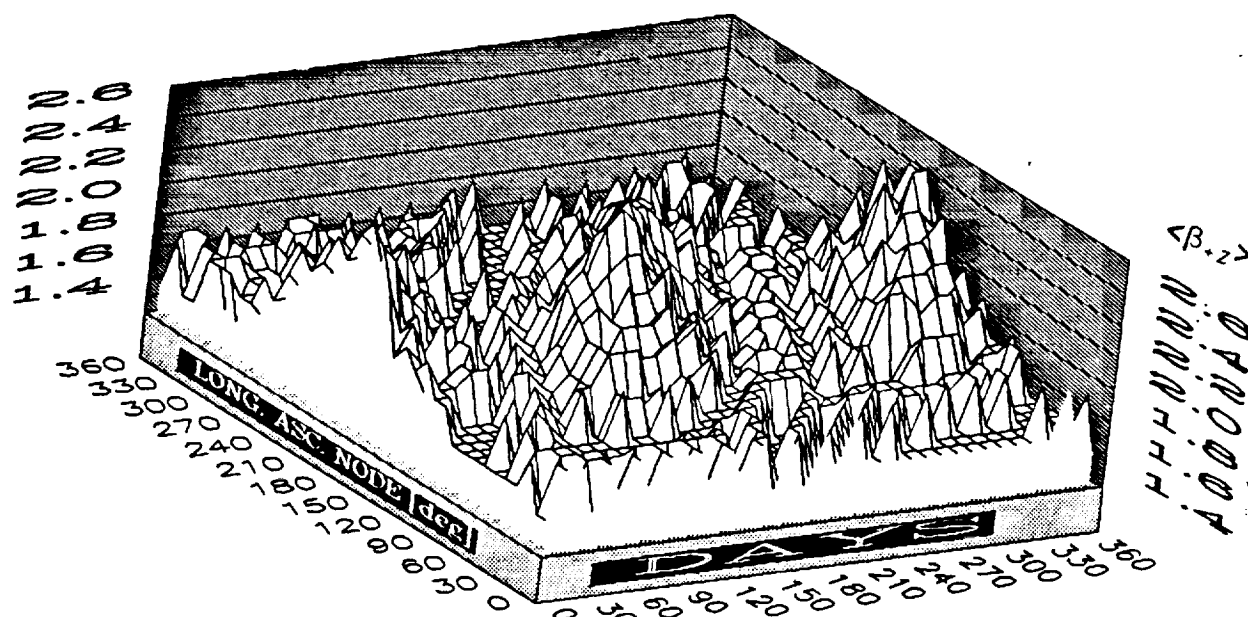


FIGURE A.7  $\langle \beta_z \rangle$

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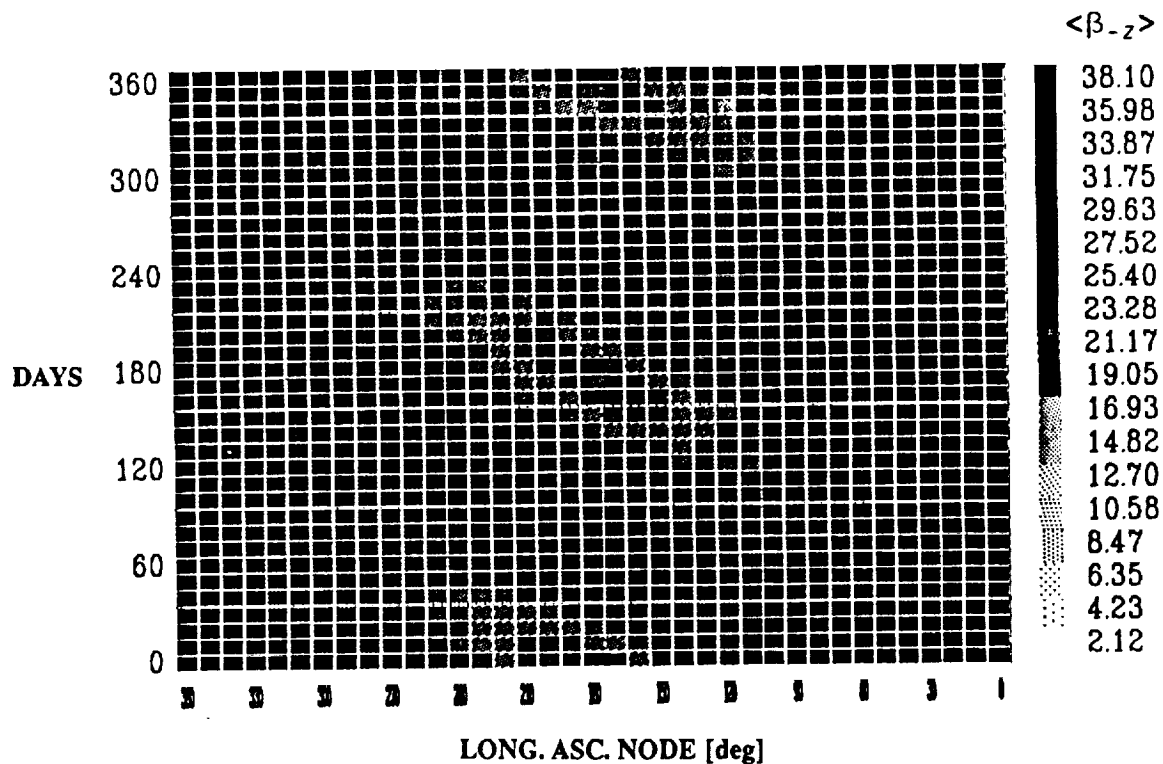
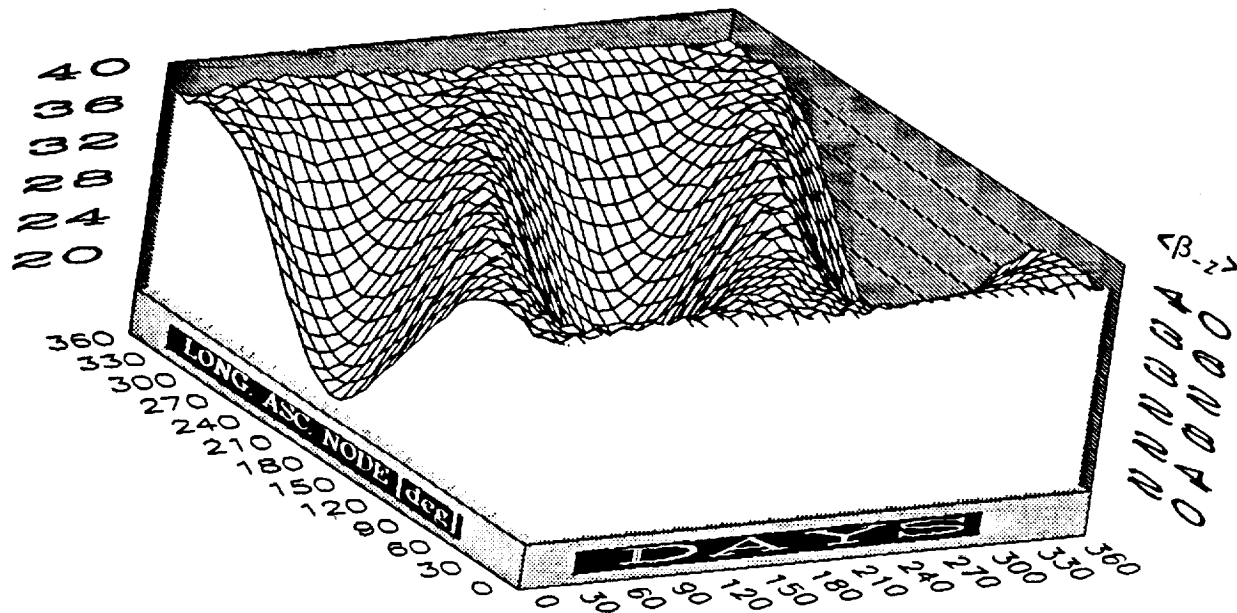


FIGURE A.8  $\langle \beta_z \rangle$

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APPENDIX B  
ATTITUDE TETHER STABILIZER

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4.2.2	Tether Stabilizing Effect
4.2.3	Remarks
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## 1.0 INTRODUCTION

The change of the baseline configuration of the Space Station from the Power Tower to the Dual Keel has been shown to be advantageous in many respects, including lower  $\mu$ -g levels in the scientific laboratories and better visibility of both Earth and Sky.

From the point of view of attitude stability and control, however, it seems that things are going worse, because of the unfavorable inertia distribution of the Dual Keel with respect to the Power Tower.

To be more specific, let us consider the Phase-1 Space Station, when the assembling is completed and the configuration is no longer subject to further changes. Considering the International Space Station (ISS) as a rigid body and using standard stability techniques, it is easy to see that the inertia ratios are such that the ISS attitude is unstable in roll-yaw and even in pitch with respect to the gravity gradient torque. This means that the Control Moment Gyros (CMG's) have to be sized not only to balance relatively small perturbing torques, but also gravity gradient itself.

In addition the physical properties of the Station indicate that this configuration is very sensitive in pitch to atmospheric drag. A four meter offset between the center of pressure and the center of gravity causes an aerodynamically induced positive torque about the pitch axis. This situation is far from favorable and appears to be a serious penalty paid to the advantages of the scientific experiments mentioned above.

Now, it is evident that the CMG's stabilization of a body with the dimensions and masses of the ISS is not a major technological problem, but the fact that gravity gradient is not a friend poses the serious problem to find other means to reach attitude stability, above all considering potential failures of the primary Attitude Control System (ACS).

These means should be passive, or semi-passive, both for simplicity and relatively low cost, and also to minimize the dynamic noise in the scientific laboratories. In this respect, information is lacking (and needed) about the spectrum of the noise originated by the CMG's and filtered by the ISS structure.

Thus, the purpose of this study is to investigate the stabilizing effect of one of these means, i.e. of ballast masses tethered to the ISS in such a way to produce additional gravity gradient torques.

## 2.0 SPACE STATION REFERENCE CONFIGURATION

In this chapter we will outline what are the basic characteristics of the Space Station for what attains the attitude control problem. As an important premise it has to be said that we relied on a minimum of rough informations [Ref.1] and on their logical consequences.

### 2.1 REFERENCE STATION PHYSICAL PROPERTIES

The phase-1 Space Station configuration is shown in Fig. 1. The Station structure consists of a transverse boom which supports the pressurized modules, a photovoltaic power system, and a thermal radiator system. The four modules are located at the center bay of the transverse boom. Tracking of the solar vector by the power generation system is provided by two sets of gimbals.

The assumed physical properties are shown in Table-1. The orbital reference frame is oriented with the Z axis along the local vertical toward Earth, the X axis along the orbital velocity and the Y axis pointing the pole of the orbit. The nominal flight configuration presents the long transverse boom aligned with the Y axis, therefore perpendicular to the orbital plane. A Torque Equilibrium Attitude (TEA) flight mode is required in order to balance the aerodynamic torque about the pitch axis, resulting in a large pitch angle.

The physical properties also indicates a misalignment of the principal inertia axes with respect to the orbital reference frame and the large drag-exposed area due essentially to the solar arrays.

### 2.2 REFERENCE ATTITUDE CONTROL SYSTEM

The Space Station attitude control is provided by a combination of Control Moment Gyros (CMG's) and chemical Reaction Control System (RCS) thrusters.



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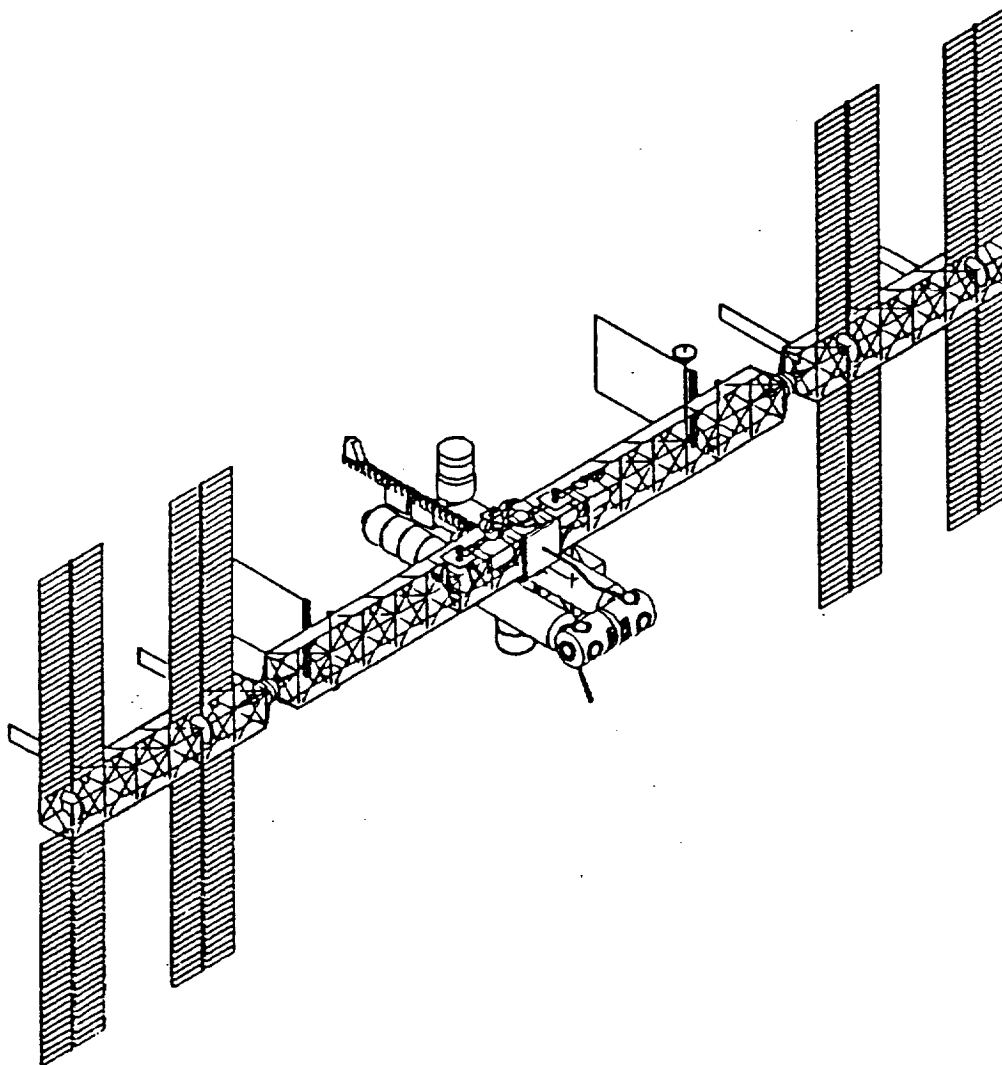


FIGURE 1 - REFERENCE SPACE STATION CONFIGURATION

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ALTITUDE = 352 KM	MASS = 219000 KG	DRAG AREA = 2169 M**2
CD = 2.3	M/CD*A = 44 KG/M**2	

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INERTIAS (KG\*M\*\*2)

IXX = 9.08 E+7	IYY = 2.54 E+7	IZZ = 1.05 E+8
IXY = 1.12 E+6	IXZ = -1.62 E+6	IYZ = -5.52 E+5

---

PRINCIPAL AXIS (DEGREES)

PSI(Z) = 1	THETA(Y) = -6.27	PHI(X) = 0.31
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C.G. LOCATION RESPECT TRUSS CENTER (M)

X = -3.16	Y = 0	Z = 3.36
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CENTER OF PRESSURE RESPECT TO C.G. (M)

X = 0	Y = -0.3	Z = -3.5
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TORQUE EQUILIBRIUM ATTITUDE (DEGREES)

PSI(Z) = 1.54	THETA(Y) = -15.52	PHI(X) = 0.4
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TABLE 1 - SPACE STATION REFERENCE PHYSICAL PROPERTIES

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### 2.2.1 Control Moment Gyros

Six CMG's are located in the transverse boom near the module cluster. The CMG's proposed are a double gimbal design with a nominal angular momentum absorption of about 5000 N\*M\*S and with a maximum torque of about 1600 N\*M when used in combination about a single axis. The CMG's are the primary effectors as momentum exchange devices during steady-state operations. They need to be periodically desaturated by the RCS and probably should be inhibited during orbit reboost, collision avoidance, and large disturbances maneuvers.

### 2.2.2 Reaction Control System

The RCS consists of four pods, with two pods located near each end of the transverse boom. The RCS design consists of 18 thrusters with thrust level of about 100 N as order of magnitude. The roll-yaw control moment arm is about 30 M and the pitch moment arm is estimated to be about 3 M. The RCS thrusters are used for CMG's desaturation, orbital reboost and as an auxiliary attitude control system in the case of CMG's inhibit or failure.

### 2.2.3 RCS Propellant Tank Farm

To implement RCS functions a hydrogen-oxygen system is utilized which provides a specific impulse of about 380 seconds. Water will be scavenged from the Orbiter fuel cells. An electrolyzer is utilized on board the Station to decompose the water back into hydrogen and oxygen and stored for use as propellant. The Station can generate fuel at a rate of about 22 Kg/Day. Current design sizing utilizes on board propellant storage tanks that have a capacity of about 1500 Kg, and two storage tanks for water that have a total capacity of 1800 Kg. The water is scavenged from the Orbiter at a rate of 600 Kg for flight.

### 3.0 STATION ATTITUDE CONTROL FEATURES

The Space Station inertia properties result in a spacecraft that is unstable in a free drift flight mode. This means that the Space Station is not able to maintain stable attitude librations around flight attitude if it is not actively controlled. In fact the inertias relationships are:

$$I_{yaw} > I_{roll} \gg I_{pitch}$$

which result in instability in pitch and a large instability in roll-yaw that are coupled.

Another problem is the impossibility, with a reasonable CMG's sizing, to maintain the Local Vertical Local Horizontal (LVLH) attitude flight mode that would be desirable for payload pointing requirements. In fact the high drag torque about the pitch axis results in a large secular and control momentum buildup that would soon saturate the CMG's for a LVLH flight mode.

The only flight mode allowed is the Torque Equilibrium Attitude (TEA). The Space Station assumes a pitch orientation able to reduce at a minimum the environmental torques, balancing aerodynamic torques by gravity gradient torques. This results in a large pitch equilibrium angle. The CMG's are required to compensate variations of the environmental torques along the orbit, and even in TEA they require to be desaturated approximately once per orbit by RCS.

#### 3.1 NOMINAL TEA FLIGHT MODE

Assuming a linearized form of Station dynamics, the Torque Equilibrium Attitude in pitch is given by:

$$3 \cdot n^2 \cdot (I_{yaw} - I_{roll}) \cdot \sin\theta = 0.5 \cdot C_d \cdot A \cdot \delta \cdot V^2 \cdot Z_p \cdot \cos\theta$$

where:

- n = mean orbital motion
- $\theta$  = pitch angle
- $C_d$  = drag coefficient
- $A^d$  = drag area assumed constant along the orbit
- $\delta$  = atmospheric density
- V = orbital velocity
- $Z_p$  = shift along Z axis of center of pressure respect to the CM

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Utilizing the Station physical properties shown in Table-1, and assuming an average atmospheric density of  $2.7 \cdot E-11 \text{ Kg/M}^3$  the TEA pitch angle results about 15.5 degrees. The TEA around roll and yaw are quite smaller as can be seen from Table-1. It has to be outlined that this static equilibrium is intrinsically unstable. In fact, the smallest perturbation is able to push away the Station from the TEA condition.

The atmospheric density is expected to vary along an orbit for the effect of the day/night heating and cooling. The CMG's are used to absorb the peak angular momentum due to the environmental torque disturbances when attitude is maintained at average TEA. Since the density along the flight path varies from orbit to orbit, TEA attitude orientation has to be adjusted periodically. Moreover the secular momentum buildup over the orbit has to be counteracted by the Reaction Control System.

With the assumption of a constant Station drag area along the orbit, the Fig. 2 shows the atmosphere density and the pitch equilibrium angle vs. the true anomaly of an orbit. The pitch equilibrium angle ranges from about -12 to -17 degrees around the average TEA of about -15 degrees.

The pitch torque required to maintain the average TEA or an offset in the range -2 to +2 degrees from average TEA is shown in Fig. 3 vs. true anomaly. The required pitch torques are comparatively small but the maximum control momentum is quite large as it can be seen in Fig. 4.

Moreover it is evident what happens in the case of shift from optimal average TEA; the secular momentum buildup along the orbit increases dramatically. Notice that in the real case and even for average TEA, a secular momentum buildup (comparatively small) is present and RCS thrusters have to be periodically used for CMG's desaturation.

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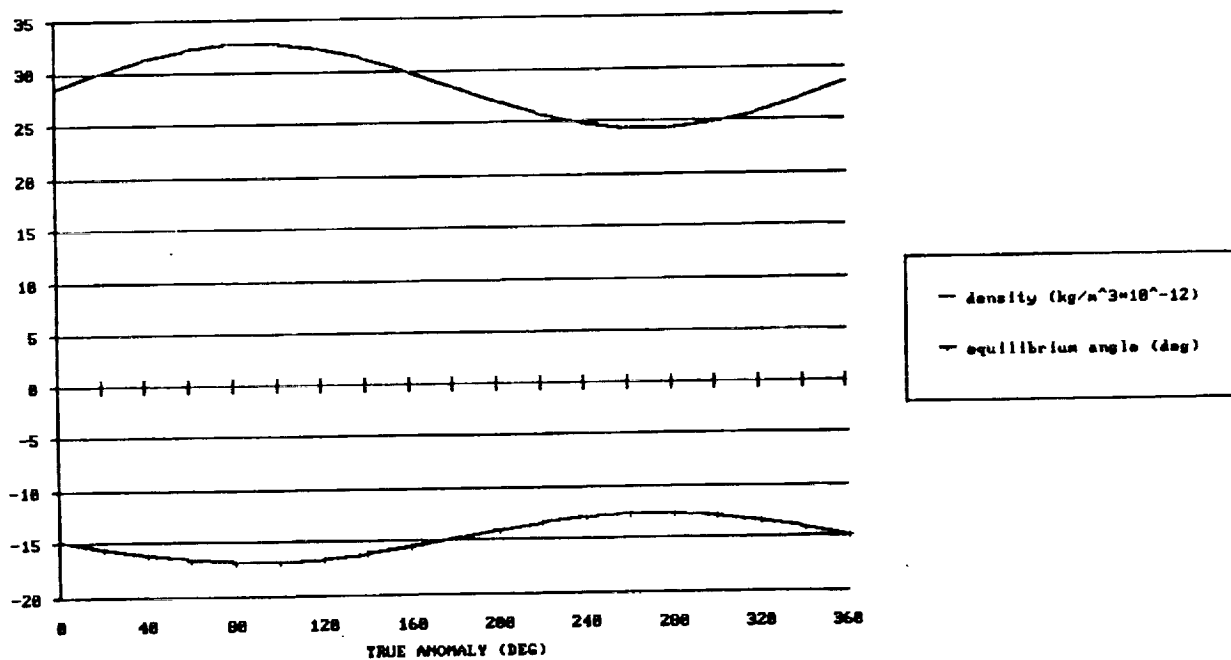
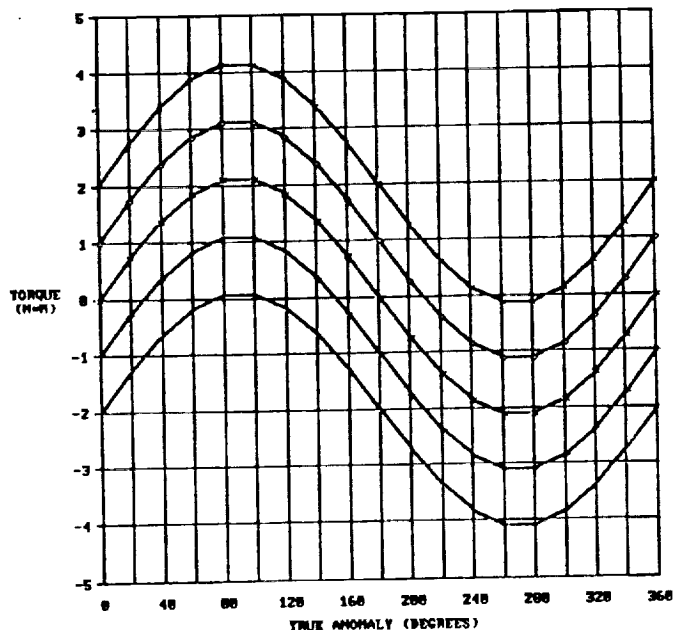


FIGURE 2 - ATMOSPHERE DENSITY AND PITCH EQUILIBRIUM  
ANGLE VS. TRUE ANOMALY



SHIFT  
FROM  
NOMINAL  
PITCH ANGLE

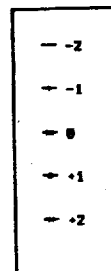
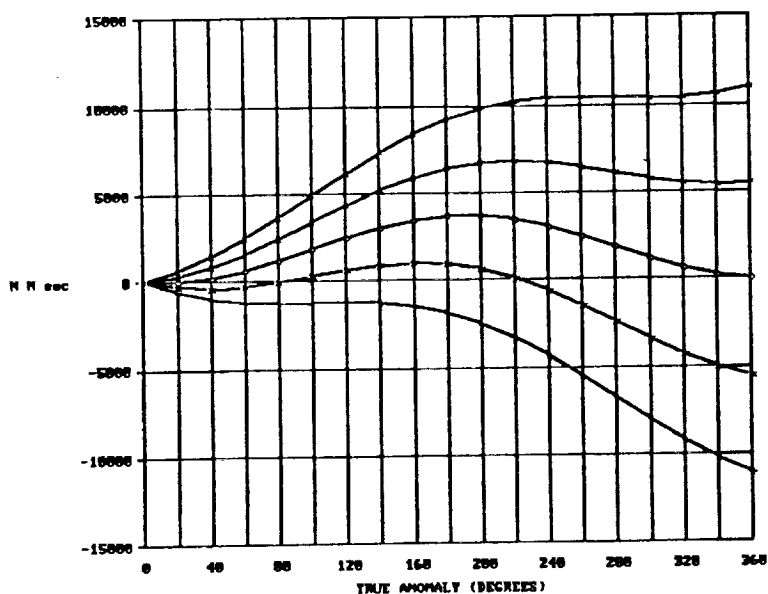


FIGURE 3 - REQUIRED PITCH CONTROL TORQUES



SHIFT  
FROM  
NOMINAL  
ANGLE



FIGURE 4 - CONTROL MOMENTUM AT TEA AND FOR OFFSETS FROM TEA

#### 4.0 PRELIMINARY ANALYSIS OF STABILITY

The addition of a tethered ballast constrained on the Space Station on a point different from the Center of Mass (CM) has a deep impact on the static and dynamics of the system.

In the following it is assumed that the tether attachment point is on a point at distance  $d$  from the center of mass along the local vertical.

#### 4.1 STATIC EFFECTS

Two are the main effects of the tether system:

- a) Tether tension causes a restoring torque when the pitch angle is made to be different from zero. The restoring torque is given by:

$$C = T \cdot d \cdot \sin(\theta)$$

where  $T$  is the tether tension. Tether tension is approximately given by:

$$T \approx 3 \cdot n^2 \cdot (M_b + 1/2 \cdot M_t) \cdot L$$

where:

$M_b$  = ballast mass  
 $M_t$  = tether mass  
 $L$  = tether length

This torque can be made as large as desired with proper tether length and ballast mass size.

When the tether system is added the resulting equilibrium angle changes, and can be made quite near to the local vertical. An useful parameter which gives an overall idea of the tether tension effect is:

$$I_d = M_b \cdot d \cdot L$$

which has the dimension of an inertia moment. To give an idea of the size of the things we are talking, to get a value of  $I_d \approx 10^8$  we can use:

$d = 10 \text{ m}$   
 $L = 6000 \text{ m}$   
 $M_b = 1700 \text{ Kg}$



The equilibrium around the pitch axis is given then by:

$$3 \cdot n^2 \cdot (I_{\text{yaw}} - I_{\text{roll}} + I_d) \cdot \sin(\theta) = C_d \cdot A / 2 \cdot \delta \cdot v^2 \cdot Z_p \cdot \cos(\theta)$$

The resulting pitch equilibrium angle for various values of  $I_d$  is shown in Fig. 5 (using the average value of  $\delta$ ).

It can be clearly seen that the pitch equilibrium angle can be kept within few degrees from the local vertical with reasonable values of  $I_d$ .

Another point worth to mention is the fact that the effects of changes in the atmospheric density can be reduced to a value as small as desired increasing the size of the tether system. In Fig. 6 the trends of the equilibrium pitch angle along the orbit are reported (they should be compared with the plots in Fig. 2). In conclusion it appears that the use of a passive tether to reduce the effects of atmospheric drag is quite effective.

- b) The presence of a single tether shifts the center of mass of the overall system increasing the level of residual gravity within the laboratories. With  $I_d$  ranging from  $50 \cdot 10^6 \text{ Kg} \cdot \text{m}^2$  to  $100 \cdot 10^6 \text{ Kg} \cdot \text{m}^2$  and  $d$  in the range from 5 to 20 m, the shift of Center of Gravity (CG) is of the order of 10 to 100 meters. These shifts can be translated in an increase of the background  $g$  level of the order of 3 to 30  $\mu\text{g}$  ( $10^{-6} \text{ g}$ ) as shown in Fig. 7. Two ways are possible to decrease the background  $g$  level (for a given value of  $I_d$ ):

I) Increasing the value of  $d$ .

II) Using two tethers deployed in opposite directions.

A third effect due to the tether is present, that is the area increase due to tether and ballast. It is easy to see that the additional area is of the order of few  $\text{m}^2$ , negligible in comparison to the overall area.

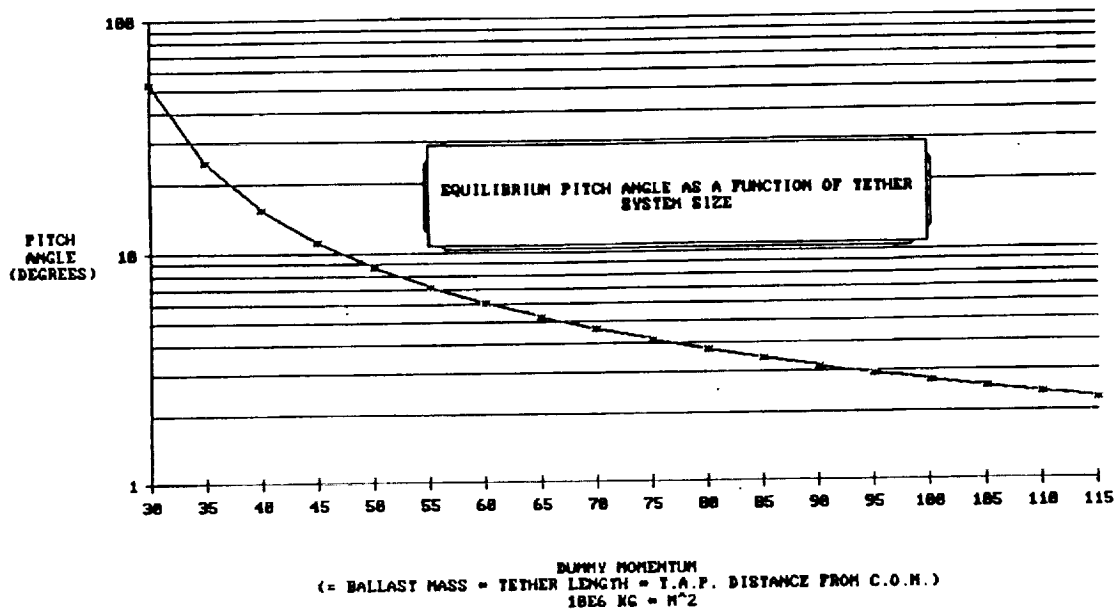


FIGURE 5 - EQUILIBRIUM PITCH ANGLES

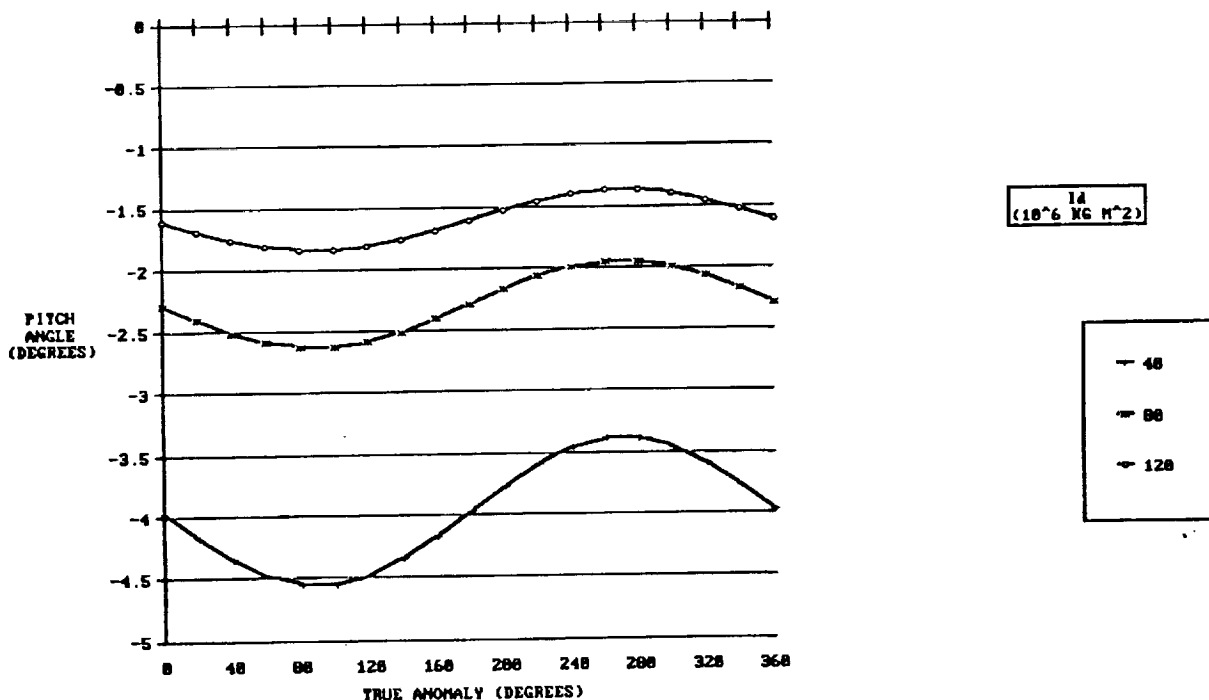


FIGURE 6 - EQUILIBRIUM PITCH ANGLES TRENDS ALONG THE ORBIT

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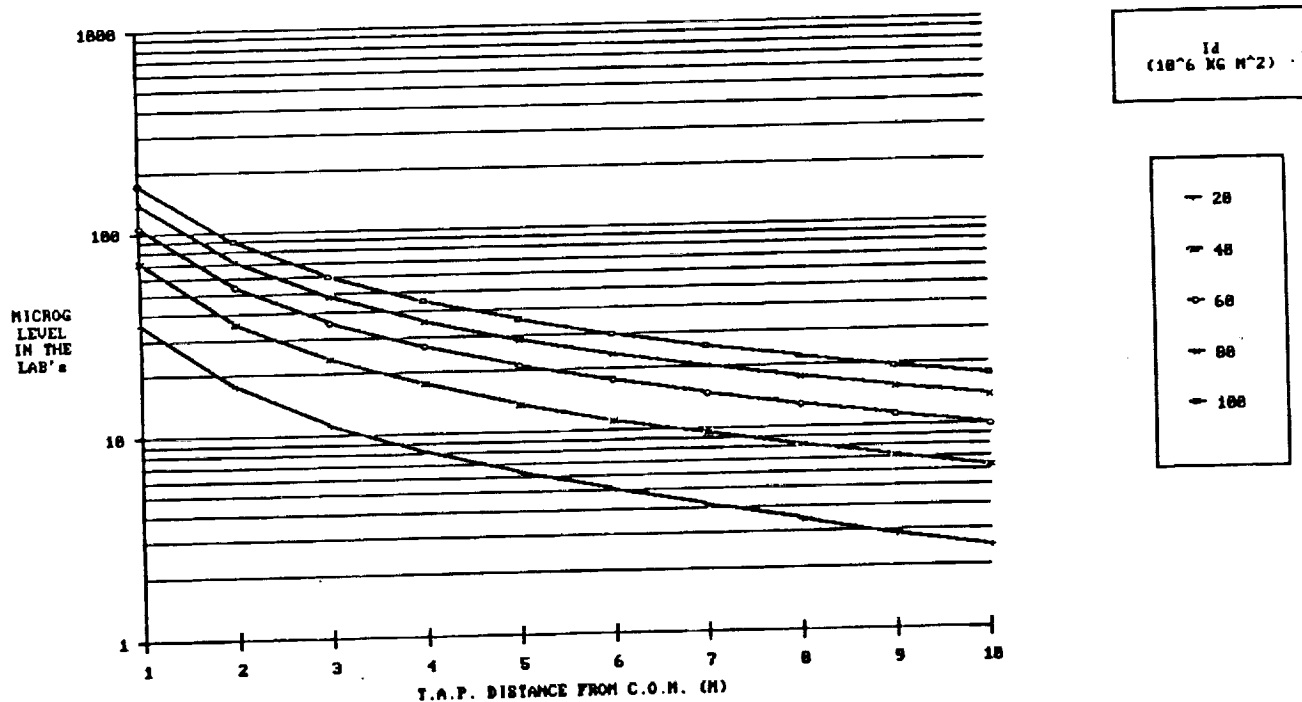


FIGURE 7 - EFFECTS OF TETHER SYSTEM ON MICRO-G LEVEL

#### 4.2 TETHERED SPACE STATION DYNAMICAL STABILITY

Over the effects on the static of the system, the tether influences also the attitude dynamics.

##### 4.2.1 Mathematical Background

The linearized equations leading the attitude dynamics of a generic body (spinning at a rate  $n$  around its pitch axis) are reported below:

PITCH EQUATION

$$I_p \ddot{\theta}_p = 0$$

ROLL EQUATION

$$I_r \ddot{\theta}_r + n(I_r + I_y - I_p) \dot{\theta}_y + n^2(I_p - I_y) \theta_r = 0$$

YAW EQUATION

$$I_y \ddot{\theta}_y - n(I_r + I_y - I_p) \dot{\theta}_r + n^2(I_p - I_r) \theta_y = 0$$

with:

$I_p$  = pitch inertia moment  
 $I_p^r$  = roll inertia moment  
 $I_r^y$  = yaw inertia moment  
 $\theta_y^p$  = pitch angle  
 $\theta_p^r$  = roll angle  
 $\theta_r^y$  = yaw angle

It is assumed that the principal axes are aligned with the pitch, roll and yaw axes.

From the start we will concentrate on analyzing only the roll-yaw equations which are decoupled from the pitch one. What we want to investigate is the stability of the motion. Basically what we do is to find out the eigenvalues of the matrix shown in Tab.2 and to verify that the real part of the roots is zero (meaning that there is not an exponential growth of the affected angle). To simplify the discussion we can use, instead of the values of  $I_p, I_r$  and  $I_y$ , the three parameters  $K_p, K_r$  and  $K_y$  where:

$$\begin{aligned} K_p &= (I_r - I_y)/I_p \\ K_r &= (I_p^r - I_y^r)/I_p^r \\ K_y &= (I_p^p - I_r^p)/I_r^p \end{aligned}$$

$$s := -\frac{\dot{S}}{n}$$

Pitch equation	$\begin{bmatrix} s^2 \cdot I_p & 0 & 0 \\ 0 & s^2 \cdot I_r + \begin{bmatrix} I_p & -I_y \\ I_y & I_p \end{bmatrix} & s \cdot \begin{bmatrix} I_y & -I_p & -I_r \\ I_y & I_p & I_r \end{bmatrix} \\ 0 & -s \cdot \begin{bmatrix} I_y & -I_p & -I_r \end{bmatrix} & s^2 + \begin{bmatrix} I_p & -I_r \\ I_p & I_r \end{bmatrix} \end{bmatrix}$
Roll equation	
Yaw equation	

TAB. 2 - FREQUENCY MATRIX OF A SPINNING BODY

Pitch equation	$\begin{bmatrix} s^2 & 0 & 0 \\ 0 & s^2 + K_r & s \cdot \begin{bmatrix} 1 - K_r \\ 1 - K_r \end{bmatrix} \\ 0 & -s \cdot \begin{bmatrix} 1 - K_y \\ 1 - K_y \end{bmatrix} & s^2 + K_y \end{bmatrix}$
Roll equation	
Yaw equation	

TAB. 3 - PARAMETRIZED FREQUENCY MATRIX OF A SPINNING BODY

Given the physical meaning of  $I_p$ ,  $I_r$  and  $I_y$  the values of the  $K$ 's are always bracketed within the range -1 and 1. In Fig. 8.a the relationship between the values of  $K_r$ ,  $K_y$  and the inertia distribution of the body is shown. Using this parametrization the frequency matrix becomes the one reported in Table-3. It is easy to see that the only condition needed to assure stability around roll-yaw is:

$$K_y K_r > 0$$

This is a known result saying that the spin axis has to be either the minimum or maximum inertia axis. A pictorial representation of this result is shown in Fig. 8.b.

Let us now consider the effect of gravity gradient only (local vertical along the yaw axis). From the expansion of the gravity field expression one can derive the equations reported below:

#### PITCH EQUATION

$$I_p \theta_p = -3 \mu / R^3 (I_r - I_y) \theta_p$$

#### ROLL EQUATION

$$I_r \theta_r = -3 \mu / R^3 (I_p - I_y) \theta_r$$

#### YAW EQUATION

$$I_y \theta_y = 0$$

In this case only the roll and pitch degrees of freedom are affected. Restricting our scope to the yaw and roll axes the stability condition is:

$$K_r > 0$$

and the relevant stability diagram is the one shown in Fig. 8.c.

Finally in the general case of the Earth-pointing satellite in circular orbit the relevant equations are those reported below (the gravity field and the orbital rate are related by the relationship  $\mu = n^2 R^3$ ).

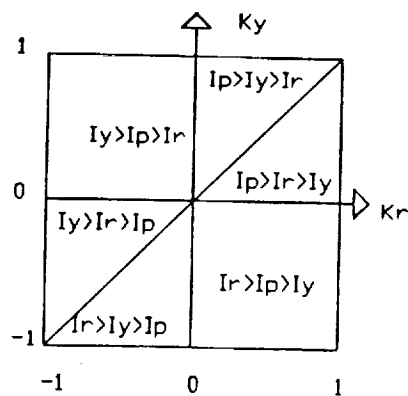


Fig. 8.a Meaning of Ky and Kr

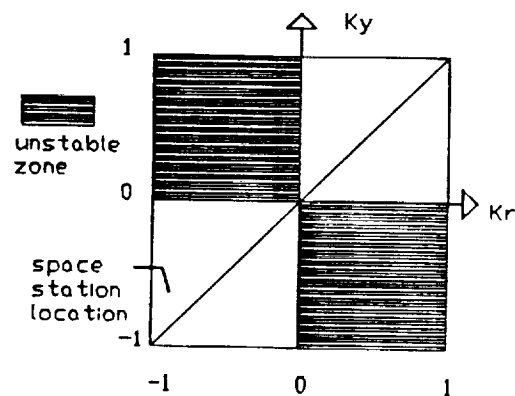


Fig. 8.b Spinning body stability diagram

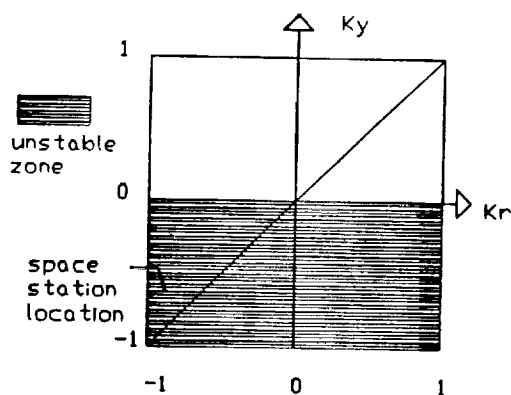


Fig. 8.c Gravity field stability diagram

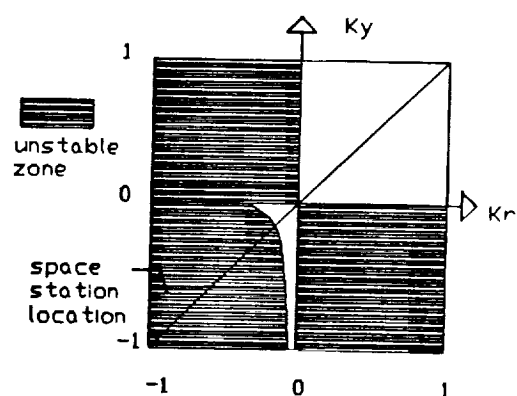


Fig. 8.d Earth pointing orbiting satellite stability diagram

#### PITCH EQUATION

$$I_p \ddot{\theta}_p = -3 n^2 (I_r - I_y) \theta_p$$

#### ROLL EQUATION

$$I_r \ddot{\theta}_r + n (I_r + I_y - I_p) \ddot{\theta}_y + 3 n^2 (I_p - I_y) \theta_r = -n^2 (I_p - I_y) \theta_r$$

#### YAW EQUATION

$$I_y \ddot{\theta}_y - n (I_r + I_y - I_p) \ddot{\theta}_r + n^2 (I_p - I_r) \theta_y = 0$$

From the equations it can be seen that if and only if the principal axes are aligned with the rotating reference frame the equilibrium is possible.

The stability conditions for the roll-yaw then become:

$$\begin{aligned} K_r K_y &> 0 \\ 1 + 3 K_r + 3 K_r K_y &> 0 \\ (1 + 3 K_r + 3 K_r K_y)^2 &> 4 K_r K_y \end{aligned}$$

The stability diagram is then the one reported in Fig. 8.d. Notice that this diagram is not the simple superposition of the ones in Fig. 8.b and Fig. 8.c. In fact there is a stability region where the stabilizing gyroscopic forces prevail over the instability due to the pure gravity field effects.

The Space Station position in the diagram is reported and its stability problem is apparent.

#### 4.2.2 Tether Stabilizing Effect

What happens when there is a tether attached to the Space Station?

The general discussion of the problem would be extremely complex, but with a few simplifying assumptions, meaningful conclusions can be drawn.

The assumptions are:

- 1) The tether motion is not affected by the motion of the Space Station.
- 2) The distance between the Tether Attachment Point (TAP) and the Space Station CM is negligible when compared to the Space Station leading size.
- 3) The shift of the system CM is small when compared to the tether length.



With these assumptions, only the static restoring torque due to tether needs to be included in the attitude equations.

As previously shown the tether tension gives rise to a restoring torque which is approximately given by:

$$C = T d \sin(\theta)$$

where T is the tether tension. Tether tension is approximately given by:

$$T \approx 3 n^2 (M_b + 1/2 M_t) L$$

With the further assumptions that the tether mass is negligible and naming:

$$I_d = M_b d L$$

the complete dynamic equations are those here reported:

PITCH EQUATION

$$I_p \theta_p = -3 n^2 (I_r - I_y + I_d) \theta_p$$

ROLL EQUATION

$$I_r \theta_r + n(I_r + I_y - I_p) \theta_y + 3n^2(I_p - I_y + I_d) \theta_r = -n^2 (I_p - I_y) \theta_r$$

YAW EQUATION

$$I_y \theta_y - n (I_r + I_y - I_p) \theta_r + n^2 (I_p - I_r) \theta_y = 0$$

The new roll-yaw stability conditions are:

$$\begin{aligned} (4 K_r + 3 \tau) K_y &> 0 \\ 1 + 3 K_r + 3 K_r K_y + 3 \tau &> 0 \\ (1 + 3 K_r + 3 K_r K_y + 3 \tau)^2 &> 4 (4 K_r + 3 \tau) K_y \end{aligned}$$

with:

$$\tau = I_d/I_r$$

For increasing values of  $\tau$  the new stability diagrams are those shown in Fig. 9.a,.b,.c,&.d. It can be seen that the effect of increasing the tether length can be translated in a shift of an enlarged stability window where the stabilizing gyroscopic forces are greater than those due to gravity.

For the reference Space Station the two boundary values of  $I_d/I_r$  are:

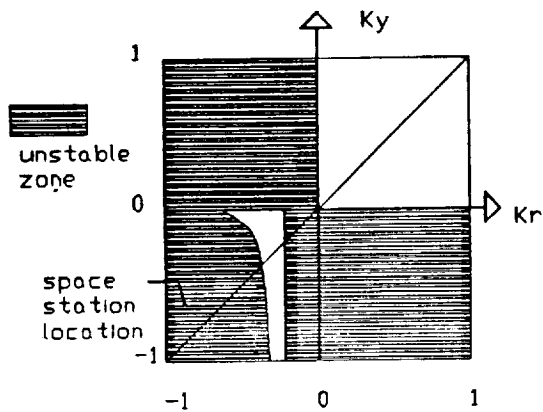


Fig. 9.a  $I_d/I_r = 0.3$

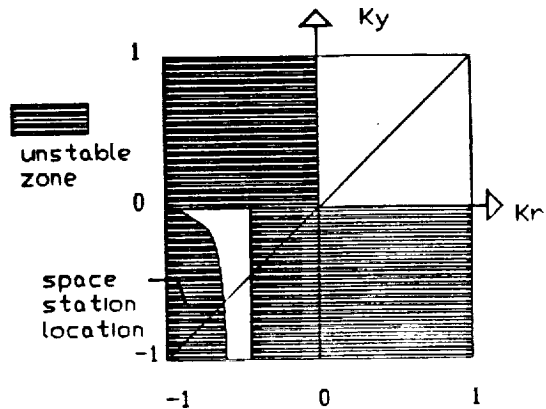


Fig. 9.b  $I_d/I_r = 0.6$

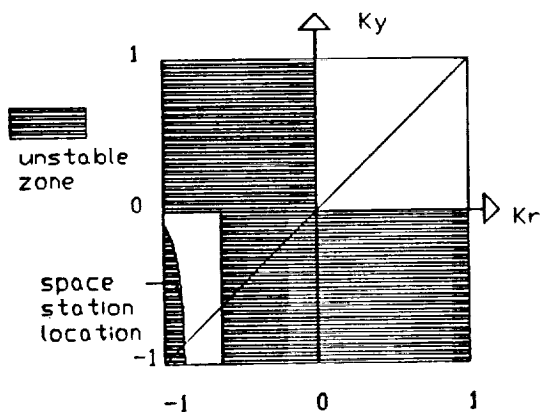


Fig. 9.c  $I_d/I_r = 0.87$   
(lower stability bound)

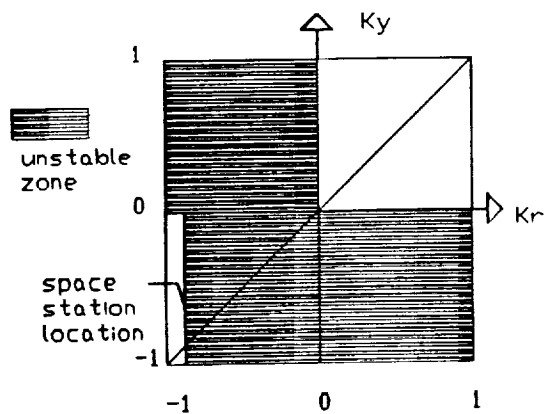


Fig. 9.d  $I_d/I_r = 1.17$   
(upper stability bound)

FIGURE 9 - TETHERED SPACE STATION STABILITY DIAGRAM

$$1.17 > I_d/I_r > 0.87$$

that is:

$$10.6 \cdot 10^7 \text{ Kg m}^2 > I_d > 7.9 \cdot 10^7 \text{ Kg m}^2$$

The relevant stability diagrams for the two limit cases are reported in Fig. 9.c and 9.d.  
The pitch motion is stable if the value of  $K_p + I_d/I_p$  is greater than zero (see the pitch equation).  
This happens for:

$$I_d > 1.42 \cdot 10^7 \text{ Kg m}^2$$

So if the system is stable in roll-yaw it is also stable in pitch.

In the prescribed range of values of  $I_d$  it is possible to find the natural frequencies of the attitude motion which are reported in Fig. 10.

#### 4.2.3 Remarks

Within the set of assumptions that we have made, we can say that it appears possible to achieve stability around the pitch and the roll-yaw axes. Notice that the tether has a direct stabilizing effect only on pitch and roll whereas the stabilization around yaw is achieved only exploiting the roll-yaw coupling due to centrifugal forces. Non linearities and/or the effects of neglected quantities require to be further investigated in order to validate this preliminary result on tether stabilizing effectiveness.

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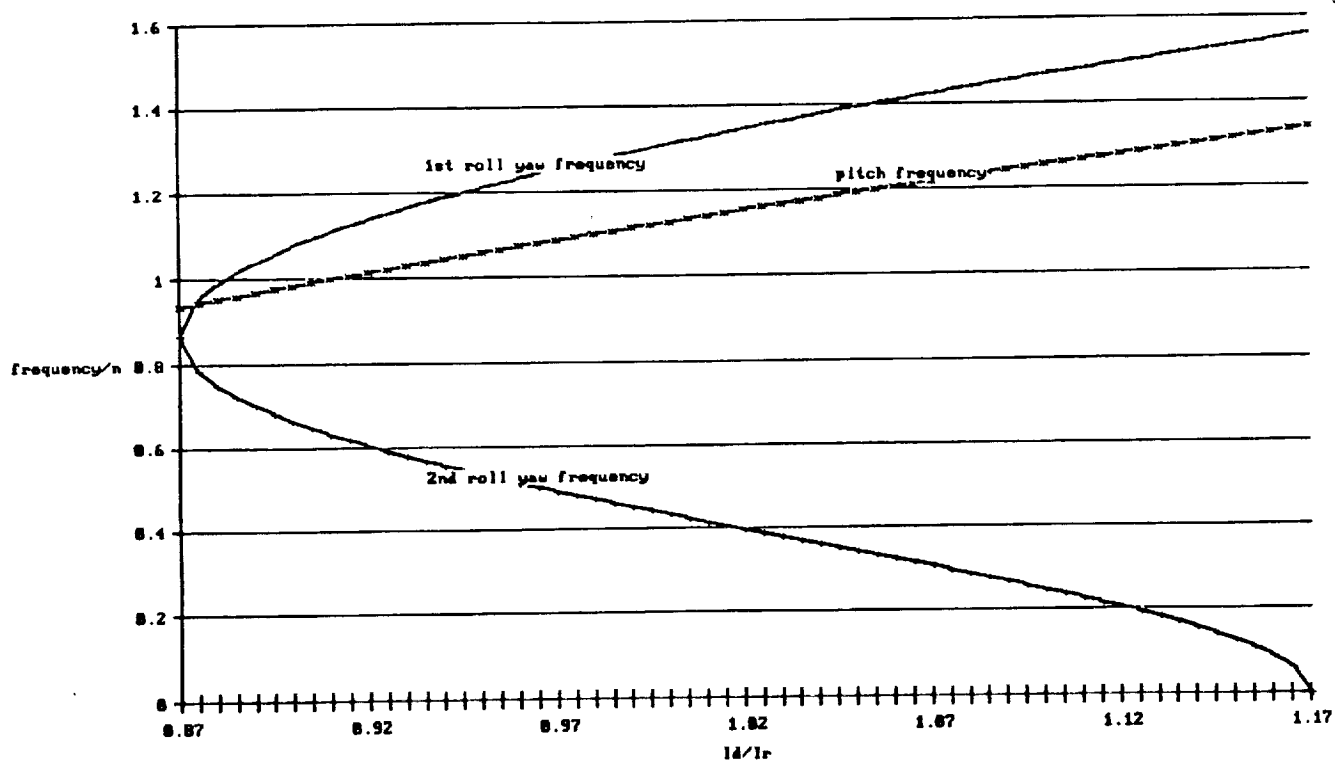


FIGURE 10 - ROLL-YAW AND PITCH FREQUENCIES AS A FUNCTION OF  $I_d/I_r$

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#### REFERENCES

- [1] L.De Ryder, P. Troutman, M. Heck, "The impact of asymmetric physical properties on large space structures", AIAA SDM issues of the International Space Station, Williamsburg, Virginia, April 1988, Paper No. 88-2486.

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#### APPENDIX C

SMITHSONIAN ASTROPHYSICAL OBSERVATORY  
PROGRESS REPORT # 7

**Analytical Investigation of  
Tethered Gravity Laboratory**

**Aeritalia Contract 8864153**

**Progress Report #7**

**For the period 16 August 1989 through 15 November 1989**

**Principal Investigator**

**Dr. Enrico C. Lorenzini**

**Co-Investigators**

**Dr. Mario Cosmo  
Mr. David A. Arnold**

**December 1989**

**Prepared for  
Aeritalia, Società Aerospaziale Italiana  
Space System Group, Torino, Italy**

**Smithsonian Institution  
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<p><b>The Smithsonian Astrophysical Observatory is a member of the Harvard-Smithsonian Center for Astrophysics</b></p>
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## **SUMMARY**

The control of the Space Station attitude by means of a tethered system has been analyzed.

Specifically, the influence of a ballast tethered to the Station upon the Station attitude dynamics when no other attitude control device is activated (e.g. wheels, thrusters), has been numerically simulated.

The results obtained are promising: the SS attitude is effectively stabilized at least for small angular displacements (i. e. within the linearity boundary). Further analysis, however, is needed to evaluate the transient response of the system for "non-linear" initial conditions.

### FIGURE CAPTIONS

- Figure 1. Schematic of SS with Tethered Ballast.
- Figures 2(a)-2(l). Attitude dynamic response of the Station acted upon by tether and gravity gradient torques. The principal body frame coincides with the geometric body frame.
- Figures 3(a)-3(n). Attitude dynamic response of the Station acted upon by tether, gravity gradient, and aerodynamic torques. The principal body frame coincides with the geometric body frame.
- Figures 4(a)-4(q). Attitude dynamic response of the Station acted upon by tether, gravity gradient, and aerodynamic torques. The principal axes of inertia are rotated by a pitch angle of  $-6.27^\circ$  with respect to the geometric body frame of the Station.

## **1.0 INTRODUCTION**

This is Progress Report #7 submitted by the Smithsonian Astrophysical Observatory (SAO) under Aeritalia Contract 8864153, "Analytical Investigation of Tethered Gravity Laboratory," Dr. Enrico C. Lorenzini, Principal Investigator. This progress report covers the period from 16 August 1989 through 15 November 1989.

## **2.0 TECHNICAL ACTIVITY DURING REPORTING PERIOD AND PROGRAM STATUS**

### **2.1 Space Station Attitude Dynamics**

At the meeting held in Torino, Italy on September 26-28, 1989, it was decided to investigate the feasibility of a tethered system for controlling the Space Station (SS) attitude in the event of a complete failure of the Station attitude control system (AOCS).

The present SS configuration has problems of attitude stability because of its mass and geometrical asymmetry. The torque equilibrium angle of the SS is around  $16^\circ$  [1] which is well above the desired values of attitude angles during the flight. Consequently, the attitude control system of the Station must be operated continually during the flight to provide the desired attitude orientation of the SS.

From a previous analysis [2] a tether system show promises for controlling the SS attitude around the three axes, in the absence of any other attitude control system.

### **2.2 Space Station Model**

The mass and orbital characteristics for the SS are as follows [1]:

Mass	219000 kg
------	-----------

Orbital Altitude	352 Km
Orbital Inclination	28.5°
Drag Area	2169 m <sup>2</sup>

#### Moments of Inertia (kg-m<sup>2</sup>)

$$\begin{aligned}
 I_{xx} &= 9.08 \times 10^7 & I_{yy} &= 2.54 \times 10^7 & I_{zz} &= 1.05 \times 10^8 \\
 I_{xy} &= 1.12 \times 10^6 & I_{xz} &= -1.62 \times 10^6 & I_{yz} &= -5.52 \times 10^5
 \end{aligned}$$

#### Center of Gravity (CG) Location with respect to Truss Center (m)

$$X_{CG} = -3.16 \quad Y_{CG} = 0 \quad Z_{CG} = 3.36$$

#### Center of Pressure (CP) Location with respect to CG (m)

$$X_{CP} = 0 \quad Y_{CP} = -0.3 \quad Z_{CP} = -3.5$$

Where the reference frame XYZ has its origin located at the SS CG and its axes are as follows:

Z-axis	along the local vertical toward the Earth;
X-axis	along the local horizontal in the orbital plane (coincides with the velocity vector for a circular orbit);
Y-axis	completes the right-handed reference frame.

This reference frame is called the Local Horizontal-Local Vertical reference frame or alternatively the rotating reference frame. When the SS has the ideal attitude orientation this reference frame coincides with the geometric body reference frame later defined.

Since the SAO computer code integrates the attitude equations in the principal body frame  $X_1X_2X_3$ , we compute the eigenvalues of the

inertia matrix. These eigenvalues are the three principal moments of inertia :

$$J_1 = 9.064 \times 10^7 \text{ kg-m}^2$$

$$J_2 = 2.538 \times 10^7 \text{ kg-m}^2$$

$$J_3 = 1.052 \times 10^8 \text{ kg-m}^2$$

We also define the geometric body reference frame with the origin at the SS CG as follows:

$Y_G$ -axis	along the truss;
$X_G$ -axis	perpendicular to the truss toward the velocity vector;
$Z_G$ -axis	completes the right handed reference frame.

The orientation of the principal reference frame with respect to the geometric reference frame is obtained by means of a 3-2-1 rotation sequence, as follows:

1<sup>st</sup> rotation around axis  $Z_G$  (yaw axis)

2<sup>nd</sup> rotation around axis  $Y_G'$  (pitch axis)

3<sup>rd</sup> rotation around axis  $X_G''$  (roll axis)

The principal reference frame is rotated with respect to the geometric body reference frame by the following angles [1]:

$$\vartheta_1(Z_G) = 1^\circ$$

$$\vartheta_2(Y_G') = -6.27^\circ$$

$$\vartheta_3(X_G'') = 0.3^\circ$$

Notice that the  $\vartheta_2$  angle is quite larger than the other two angles.

As per Aeritalia's directions, we adopted the following values for the tether system (see Fig. 1):

Tether Length	6000 m
Tether Diameter	0.008 m
Tether Mass	434 Kg
Ballast Mass	1413 kg
Distance between tether attachment point and SS CG (along $Z_G$ -axis)	10 m

The SS is assumed to be a rigid structure and the attitude equations of motion are integrated in the principal reference frame. Consequently, the Euler's equations have the classical form whereby the products of inertia are equal to zero.

The torques acting on the Station, modelled in our computer code, are the gravity gradient torque, the aerodynamic torque (which is non-zero because CP does not coincide with CG), and the torque produced on the SS by the tether tension. We call the latter "tether torque". It is worth reminding that the tether tension is also affected by the environmental perturbations acting upon the system, which are described in the previous progress reports.

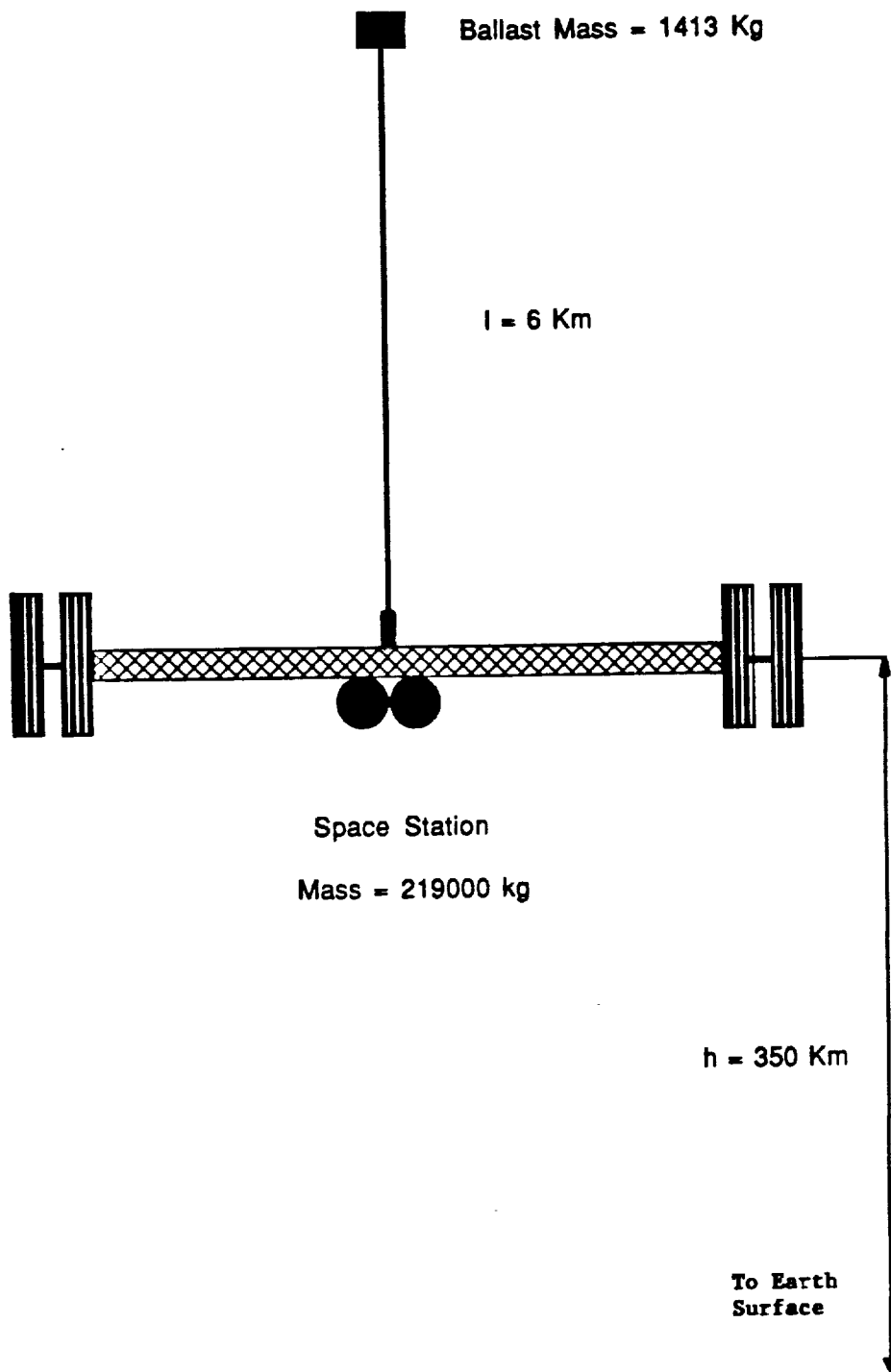


Figure 1.

### 2.3 Linearized Stability Analysis

The linearized equations of rotational motion of the SS tethered to a ballast aligned the local vertical are as follows:

$$\ddot{\theta} + 3 \Omega^2 (r d m_E / J_2 + k_2) \theta = 0 \quad (1.1)$$

$$\ddot{\phi} - (1 - k_1) \Omega \dot{\psi} + \Omega^2 (3 r d m_E / J_1 + 4 k_1) \phi = 0 \quad (1.2)$$

$$\ddot{\psi} + (1 - k_3) \Omega \dot{\phi} + \Omega^2 k_3 \psi = 0 \quad (1.3)$$

where:

$$k_1 = (J_2 - J_3) / J_1 \quad k_2 = (J_2 - J_1) / J_2 \quad k_3 = (J_1 - J_3) / J_3$$

In eqns (1)  $\Omega$  is the orbital rate,  $\theta$ ,  $\phi$ , and  $\psi$  the pitch, roll, and yaw angles respectively,  $m_E = m_B + m_T/2$  the equivalent mass,  $m_B$  and  $m_T$  the ballast and the tether mass respectively,  $d = l + r$  where  $l$  is the tether length, and  $r$  is the distance between the SS CG and the tether attachment point.

The pitch equation is independent (this is true for small angles only) while the roll and yaw equations are coupled.

After Laplace transforming eqns (1.2) and (1.3) for null initial conditions we obtain the following algebraic equations:

$$(s^2 + c_{11}) [\phi] + c_{13} [\psi] = 0 \quad (2)$$

$$c_{31} [\phi] + (s^2 + c_{33}) [\psi] = 0$$

By equating to zero the determinant of eqns (2) we obtain the characteristic equation as follows:



$$s^4 + (c_{11} + c_{33} - c_{13} c_{31}) s^2 + c_{11} c_{33} = 0 \quad (3)$$

where

$$\begin{aligned} c_{11} &= \Omega^2 \left( 4k_1 + 3 \frac{rdm_E}{J_1} \right) \\ c_{33} &= \Omega^2 k_3 \\ c_{13} &= \Omega (1 - k_1) \\ c_{31} &= \Omega (k_3 - 1) \end{aligned} \quad (4)$$

After applying the Routh-Hurwitz criterion to eqn (3) we obtain the stability conditions as follows [2]:

$$b_2 > 0 \quad \text{and} \quad b_1 > 2\sqrt{b_2} \quad (5)$$

where

$$\begin{aligned} b_1 &= 1 + k_1 k_3 + 3\left(k_1 + \frac{rdm_E}{J_1}\right) \\ b_2 &= k_1 k_3 + 3\left(k_1 + \frac{rdm_E}{J_1}\right) k_3 \end{aligned}$$

For  $k_3 < 0$  (i.e. for the current Station design) and after defining  $\tau = drm_E/J_1$ , eqns (5) yield

$$\tau < -\frac{4}{3}k_1; \quad \tau < \tau_1 \quad \text{and} \quad \tau > \tau_2 \quad (6)$$

where

$$\tau_1 = -f - \sqrt{\Delta}; \quad \tau_2 = -f + \sqrt{\Delta} \quad (7)$$

and

$$\Delta = f^2 - g$$

$$f = \frac{1}{3}(1 + k_1 k_3 + 3k_1 - 2k_3)$$

$$g = \frac{1}{9}[(1 + k_1 k_3 + 3k_1)^2 - 16k_1 k_3]$$

From eqn (1.1) the pitch motion is stable if

$$d r m_E > J_3 - J_1 \quad \text{or} \quad \tau > J_3/J_1 - 1 \quad (8)$$

For the current SS design

$$k_1 = -0.88; \quad k_2 = -0.57; \quad k_3 = -0.62 \quad (9)$$

and the stability conditions, given by eqns (6), for the roll-yaw motion yield

$$0.87 < \tau < 1.17 \quad (10)$$

while for the stability of the pitch motion we obtain from eqn (8)

$$\tau > 0.16 \quad (11)$$

Since Aeritalia's values for the tethered system are  $l = 6$  km and  $r = 10$  m, eqns (10) and (11) translate respectively into

$$1314 \text{ kg} < m_E < 1768 \text{ kg} \quad \text{roll-yaw} \quad (12.1)$$

$$m_E > 241 \text{ kg} \quad \text{pitch} \quad (12.2)$$

Consequently, the value of  $m_E = 1630$  kg, selected by Aeritalia, satisfy eqns (12) and we should expect a stable attitude motion of the SS around the three axes at least for small angles.

## 2.4 Numerical Results

The first two simulations were run by assuming that the SS principal axes coincide with the  $X_G Y_G Z_G$  frame. Conversely, for the third simulation we assume that the principal reference frame is rotated with respect to the geometric reference frame by a  $-6.27^\circ$  pitch angle. The other smaller angular displacements have been neglected.

Furthermore, all the environmental perturbations (i.e. thermal, gravitational, and aerodynamical) modelled in the SAO computer code are acting upon the system except for Run #1 as explained below.

In brief the simulation runs presented in this report are as follows:

- |               |   |
|---------------|---|
| <b>Run #1</b> | SS attitude dynamics acted upon by tether and gravity gradient torques. Null initial attitude angles. The principal body frame coincides with the geometric body frame; |
| <b>Run #2</b> | same as Run #1 except for the aerodynamic torques and forces which are now activated;   |
| <b>Run #3</b> | same as Run #2 except for the principal body frame which is now rotated by a $-6.27^\circ$ pitch angle with respect to the geometric body frame.                        |

Figures 2(a)-2(l) show the dynamic response of Run #1. Figure 2(a) shows the in-plane and out-of-plane angles of the tether with respect to the local vertical.

Figures 2(b), 2(c), and 2(d) show the pitch, roll, and yaw angle, respectively. All three angles are bounded and stable. Note that the pitch motion is uncoupled from the yaw and roll motions which is to

be expected for small amplitudes of the attitude angles. Moreover, the pitch angle follows the tether in-plane angle.

Figures 2(e), 2(f), and 2(g) depict the three components of the gravity gradient torques, while figures 2(h) and 2(i) show the two components of the tether torque, whereas the component about the yaw angle is null. Obviously, the attitude angles are modulated according to the frequency content of the perturbation torques. The tether torque has a peak value of about 10 N-m about axis-1, while the gravity gradient has a peak value of 8 N-m about the same axis.

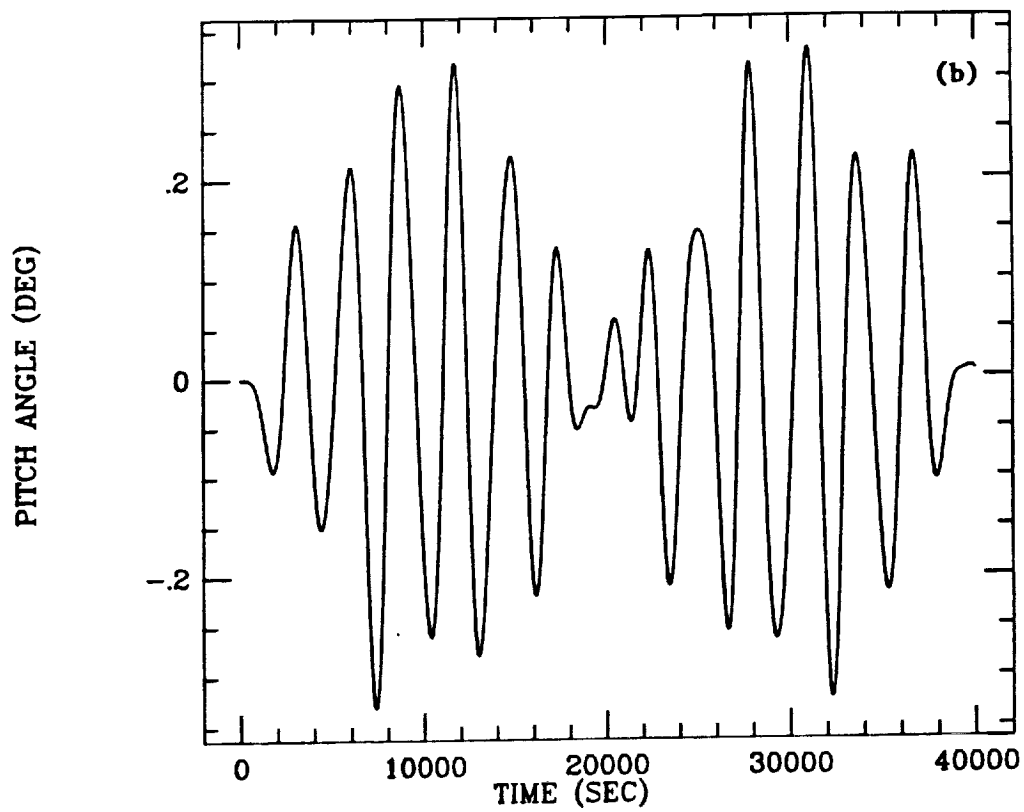
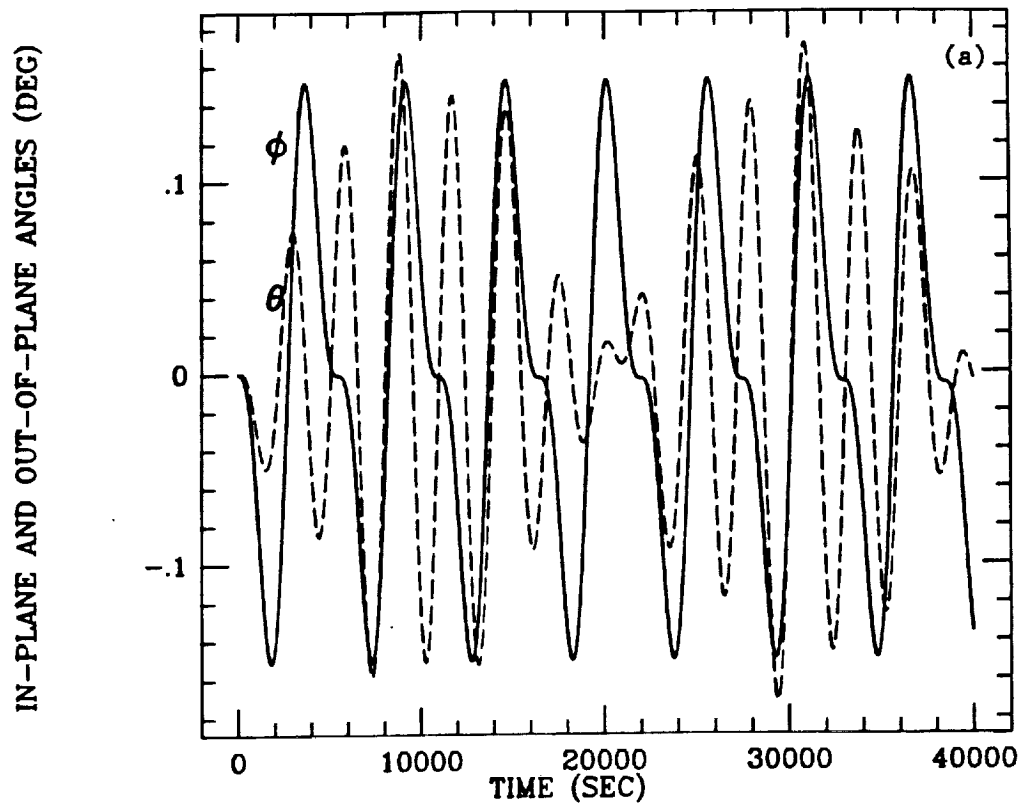
Figure 2(j), 2(k), and 2(l) show the total torque components acting around each axis.

Figures 3(a)-3(n) show the dynamic response of Run #2. Because of the drag acting upon the station, the pitch angle [Fig. 3(a)] does not oscillate any longer around the zero value and it also exhibits a harmonic component at the frequency of the pitch natural oscillation superimposed on the frequency of the forcing term. The roll and the yaw, however, show the same behavior of Run #1. Even in this case the pitch is uncoupled from the roll and yaw motions. More important, the three attitude angles are still bounded and stable notwithstanding the drag torques.

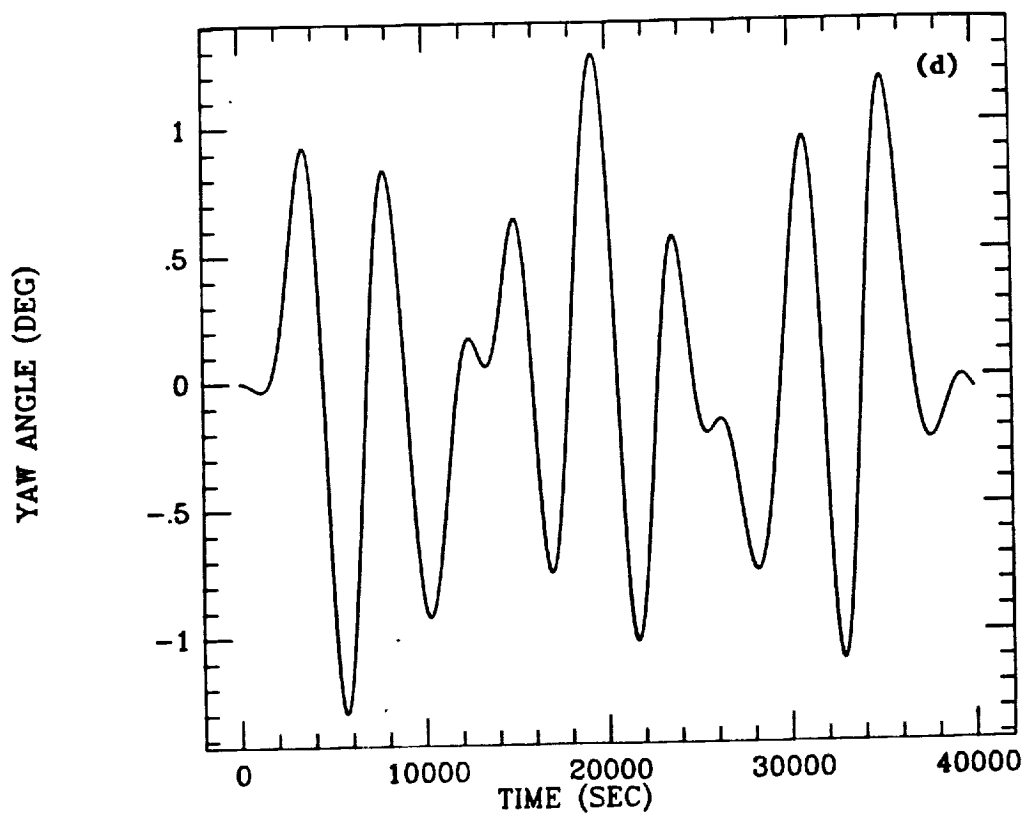
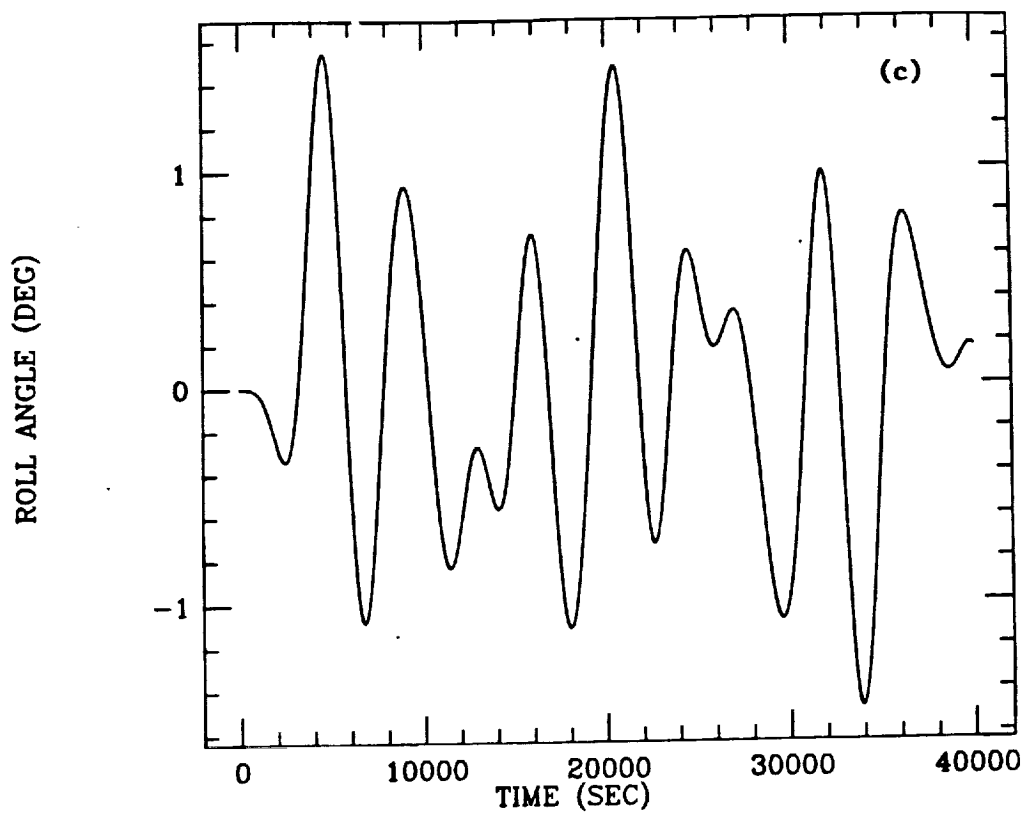
Also in this case the tether torque and the gravity gradient torque are stronger about axis-1, while the aerodynamic torque is stronger around axis-2 (peak value equal to 5 N-m). The remaining figures are self-explanatory.

Figures 4(a)-4(q) show the dynamic response of Run #3. For easier comparison with the previous runs we plot as attitude angles the angular displacements of the geometric body frame with respect to the Local Horizontal-Local Vertical reference frame.

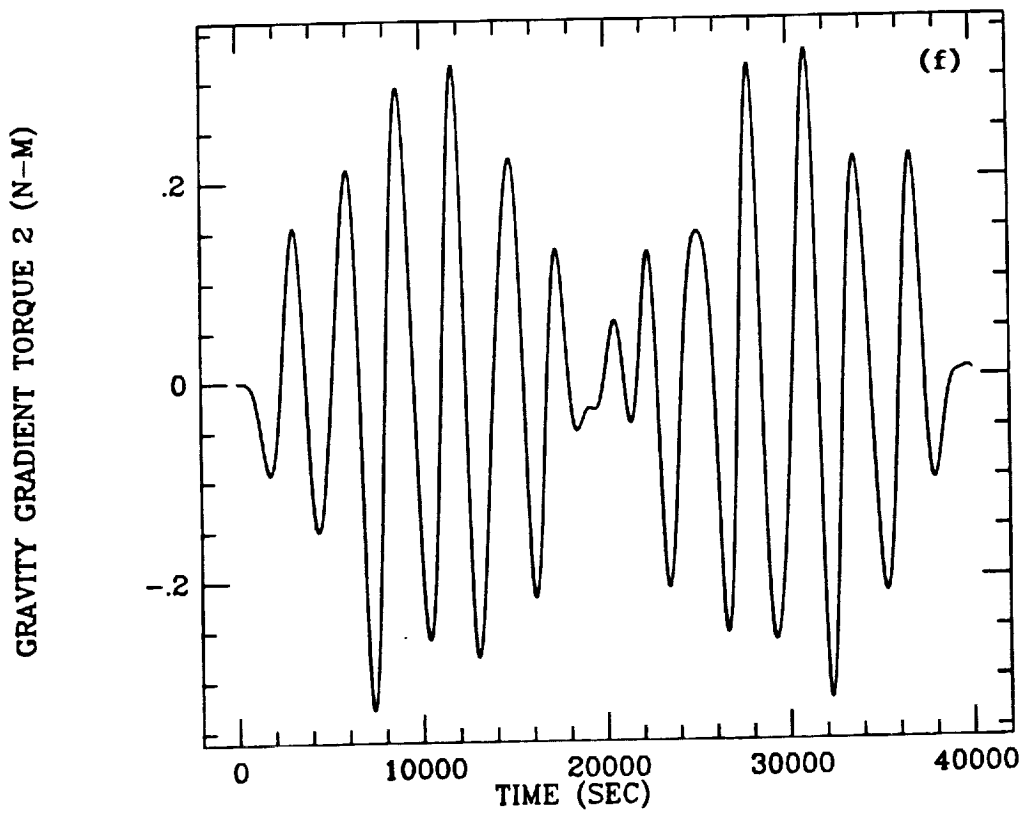
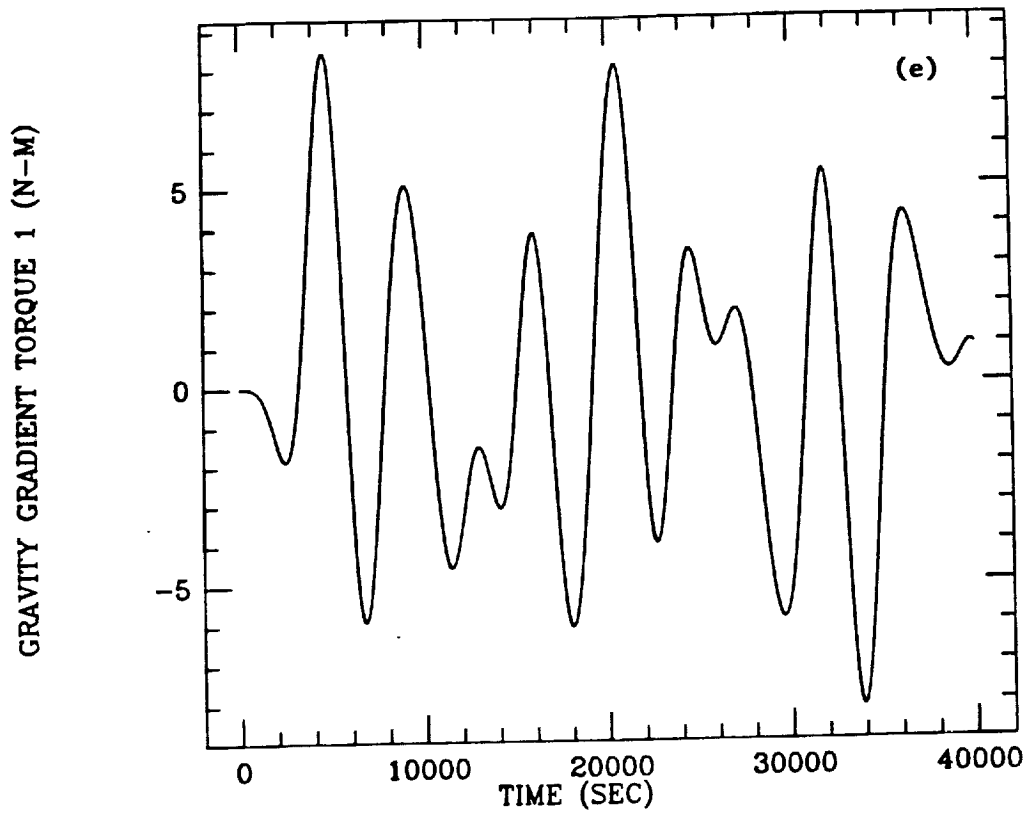
No major differences can be noticed when the results of this run are compared to Run #1. The dynamic response is stable: the tether system is able to stabilize the attitude of the Station around the three



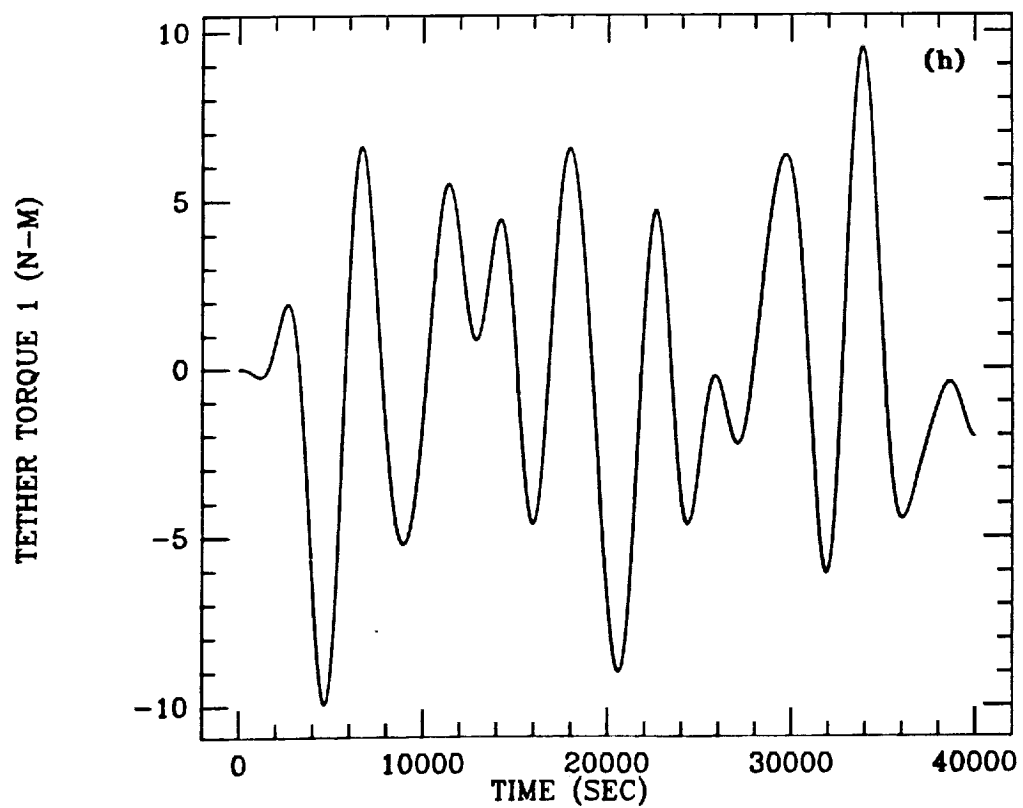
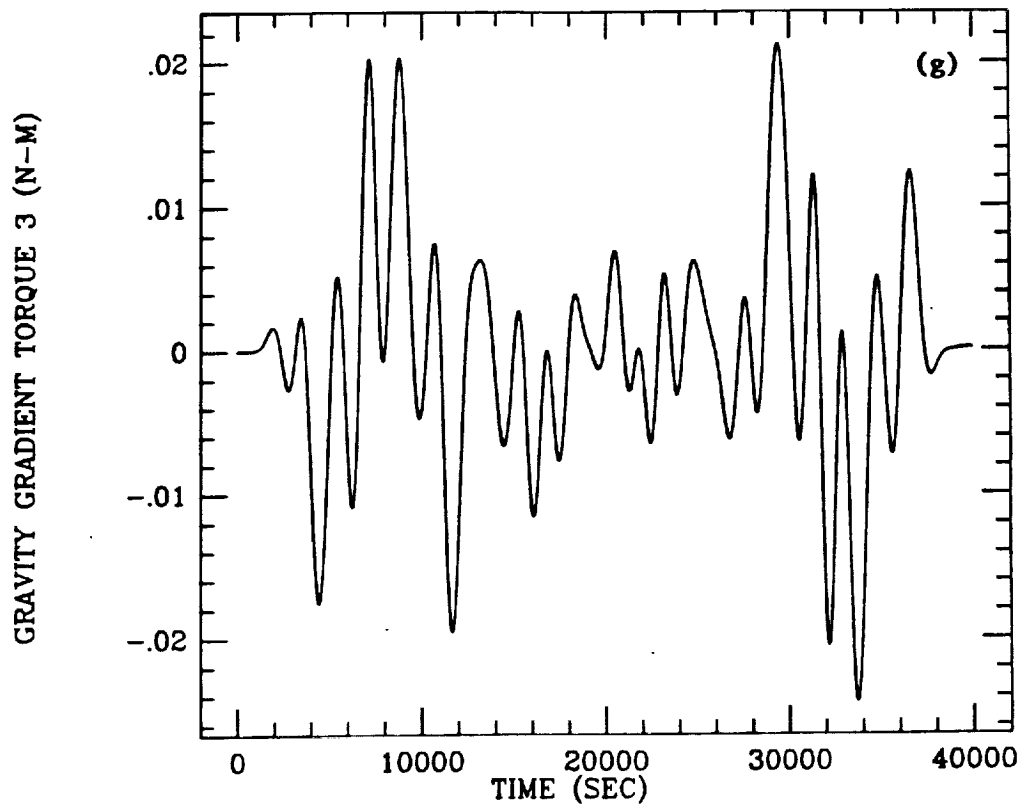
Figures 2a and 2b.



Figures 2c and 2d.

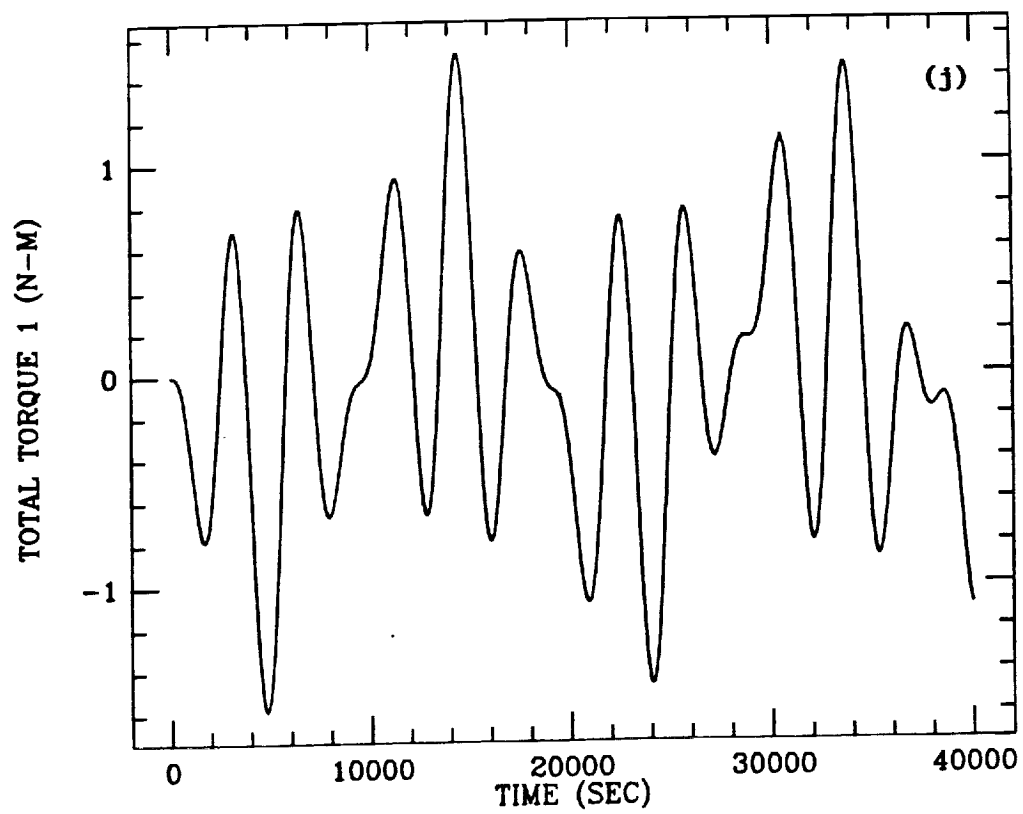
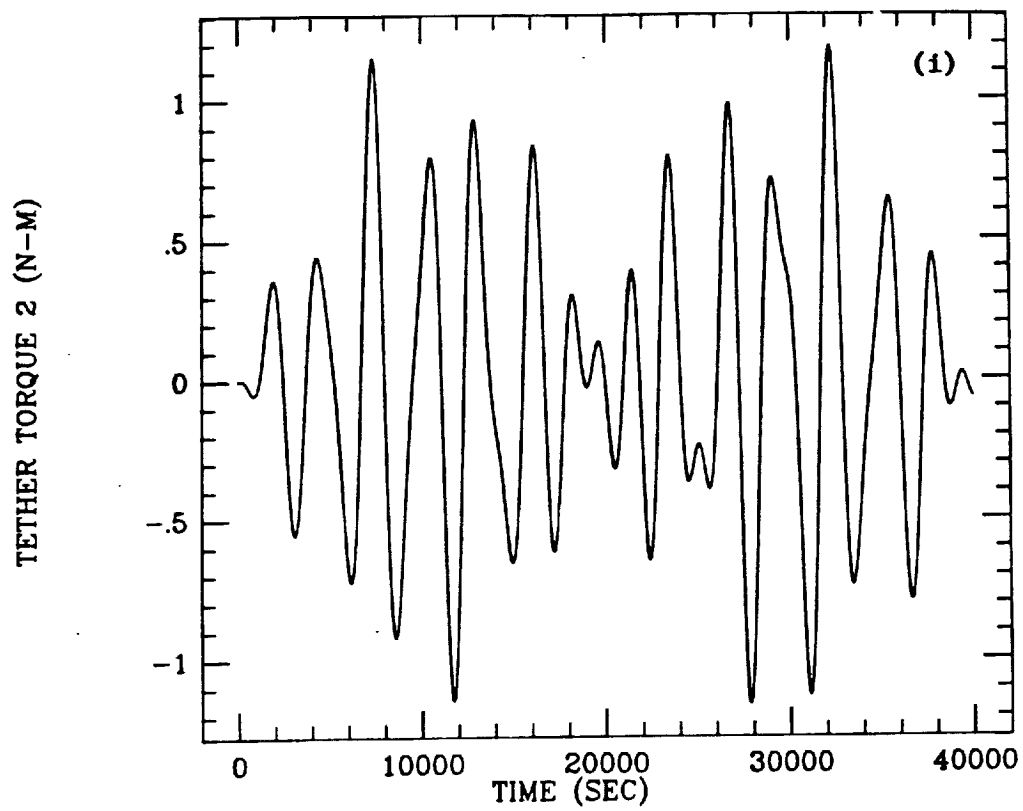


Figures 2e and 2f.

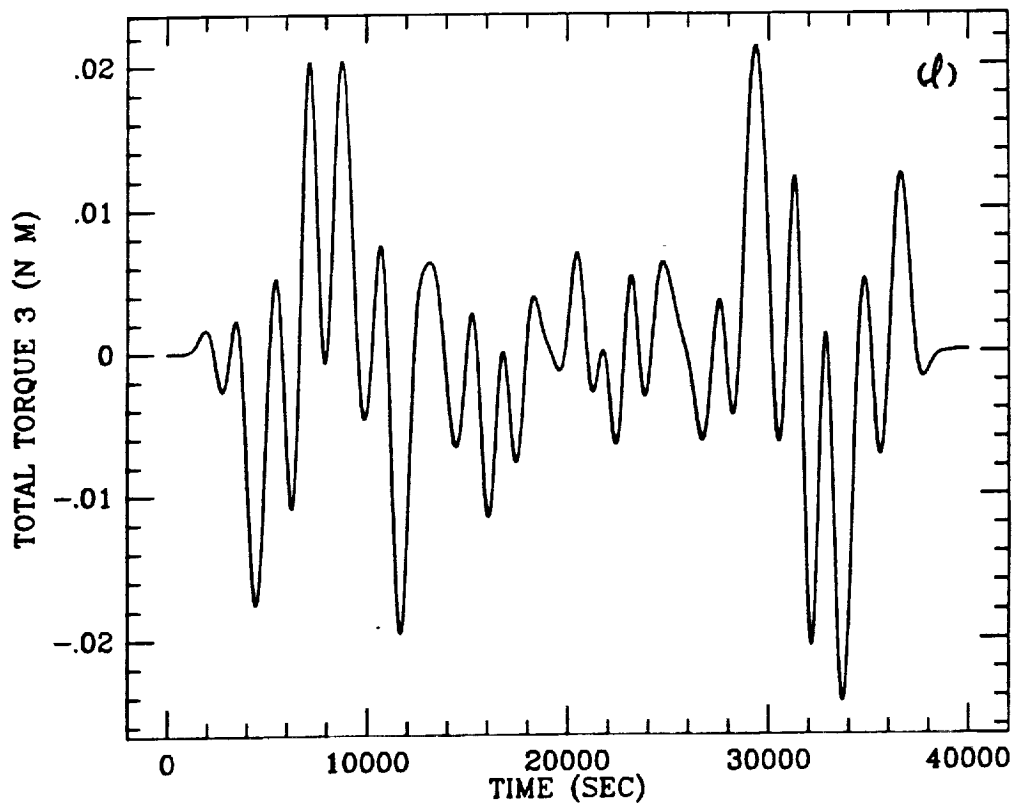
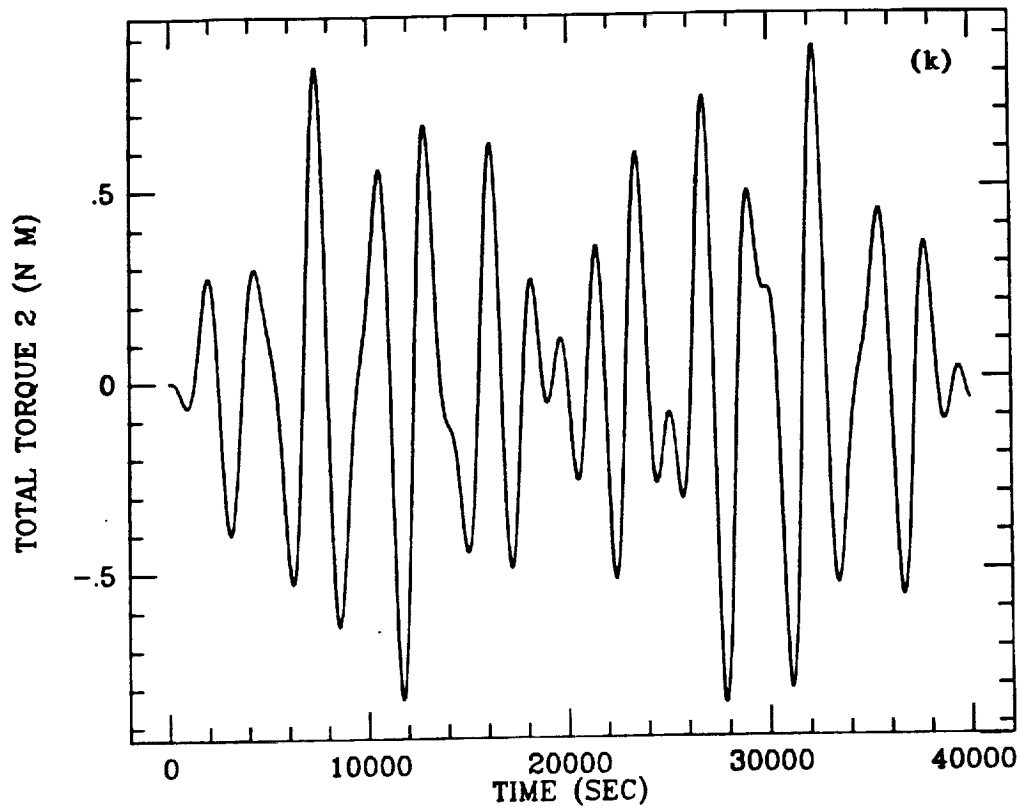


Figures 2g and 2h.

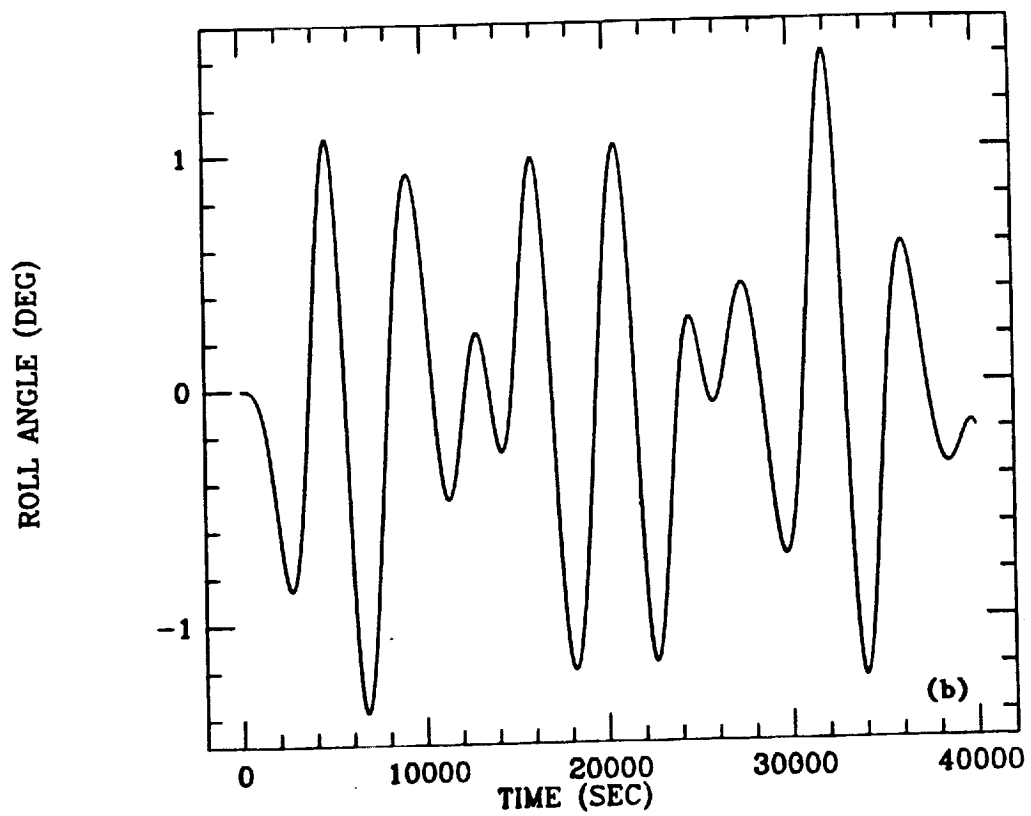
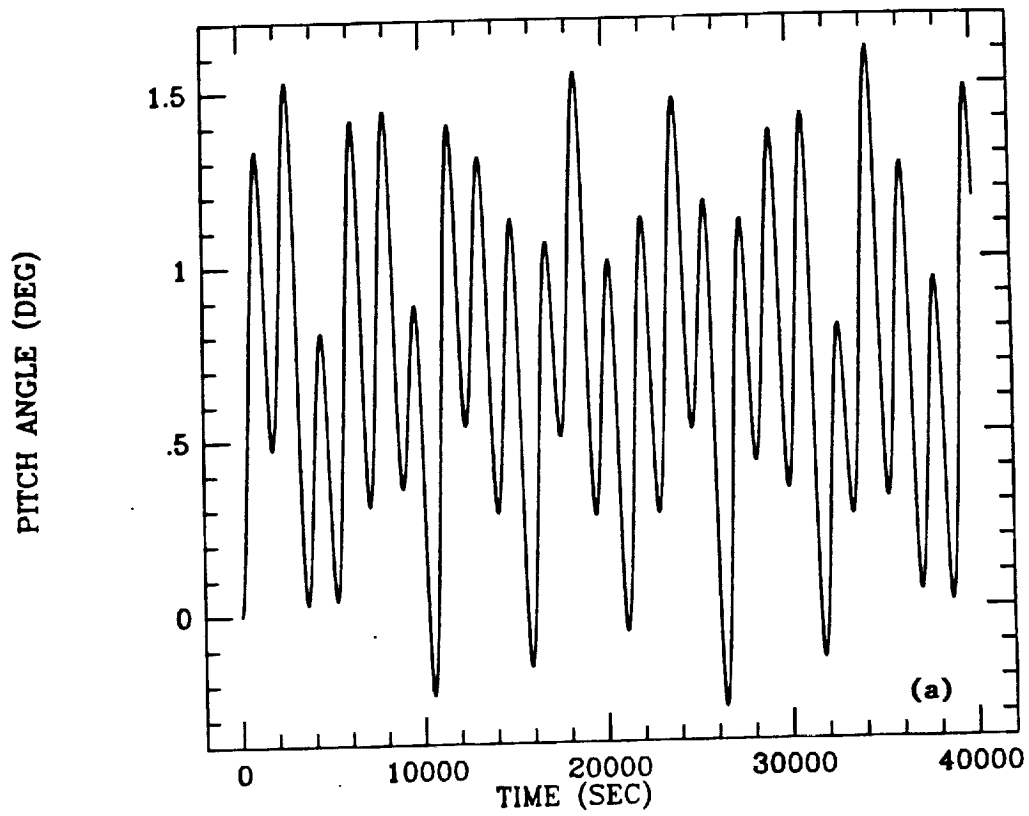




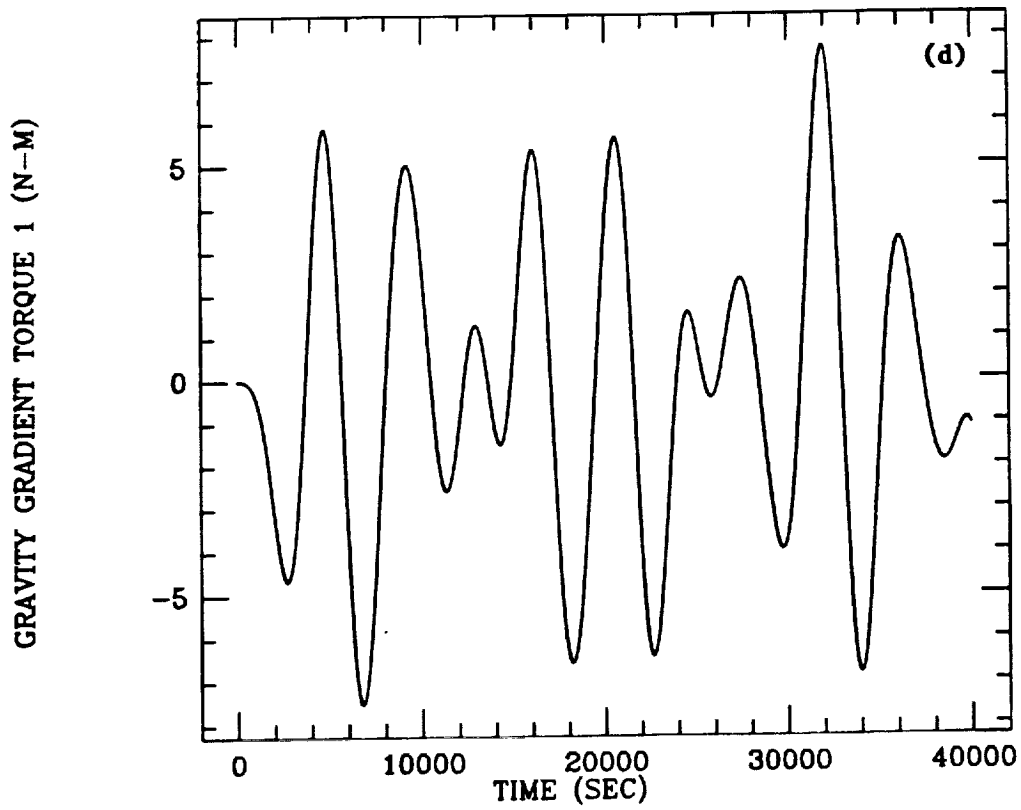
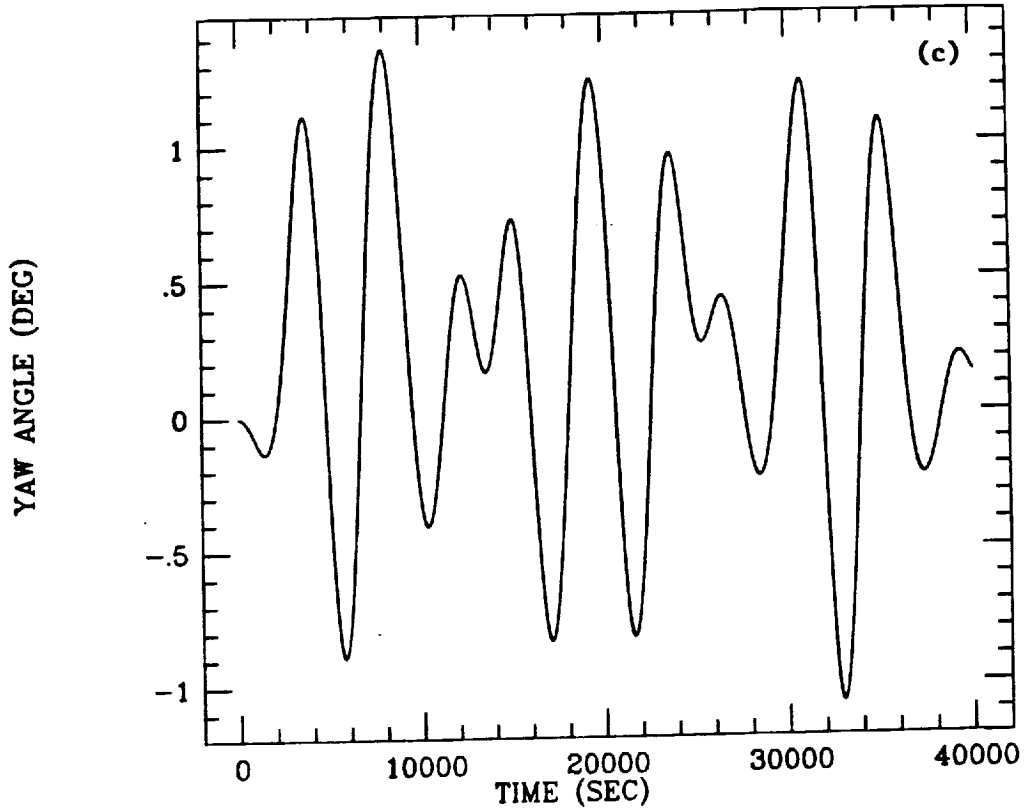
Figures 2i and 2j.



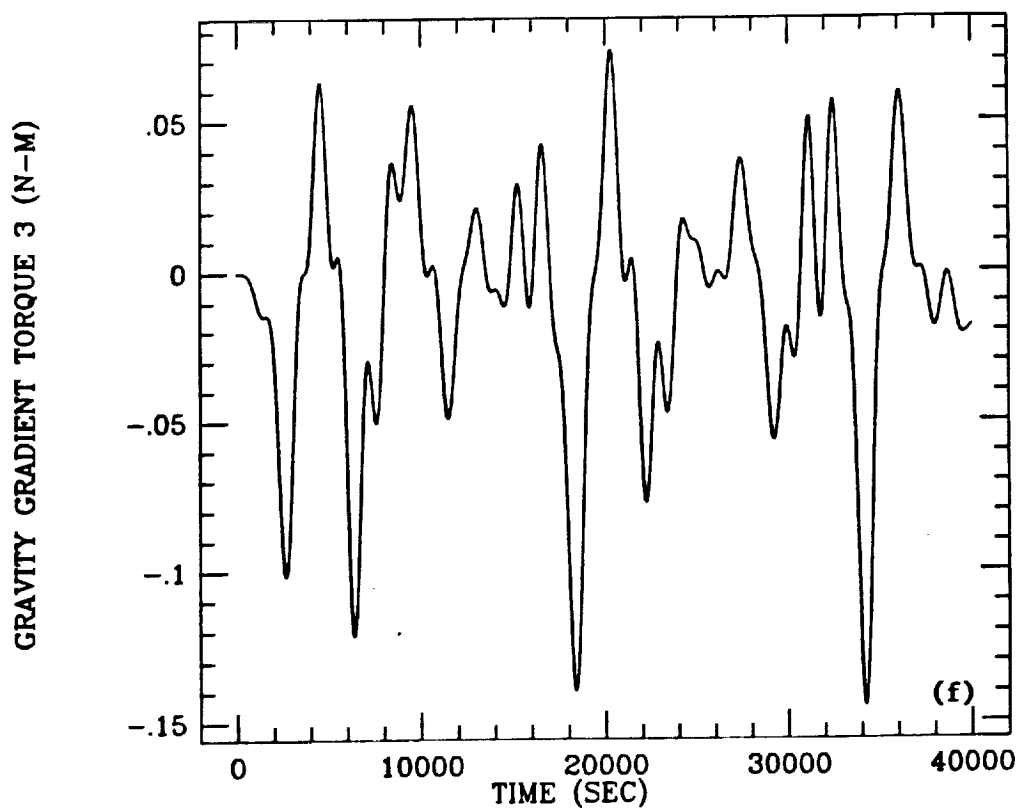
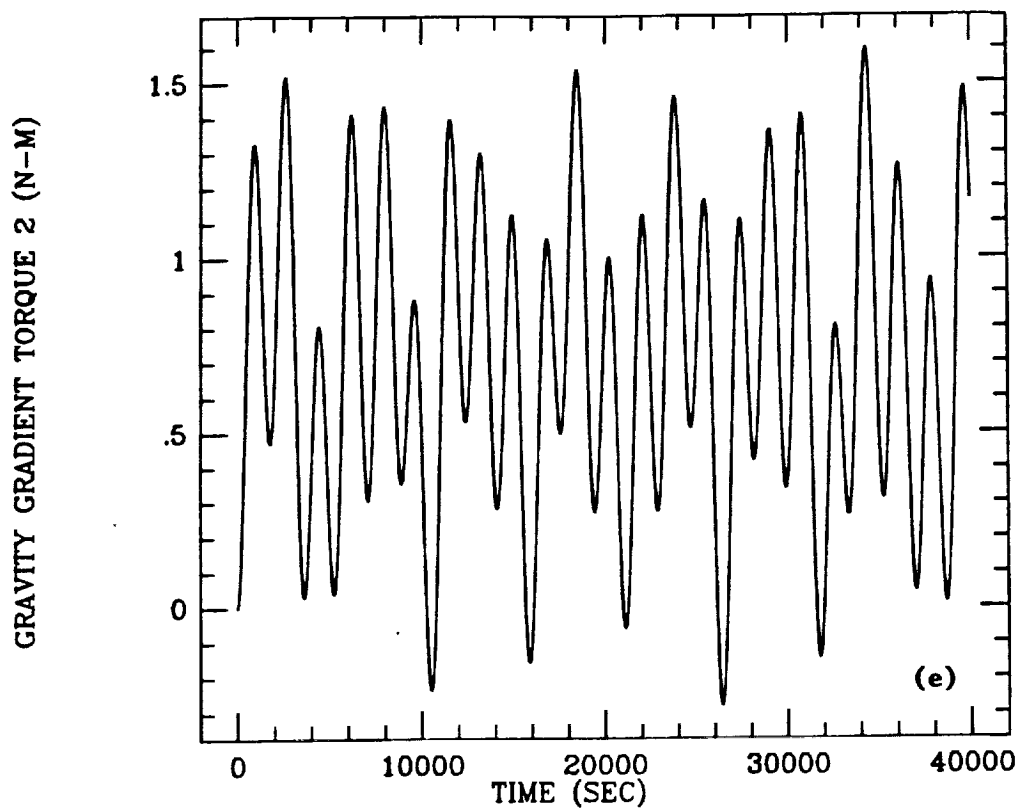
Figures 2k and 2l.



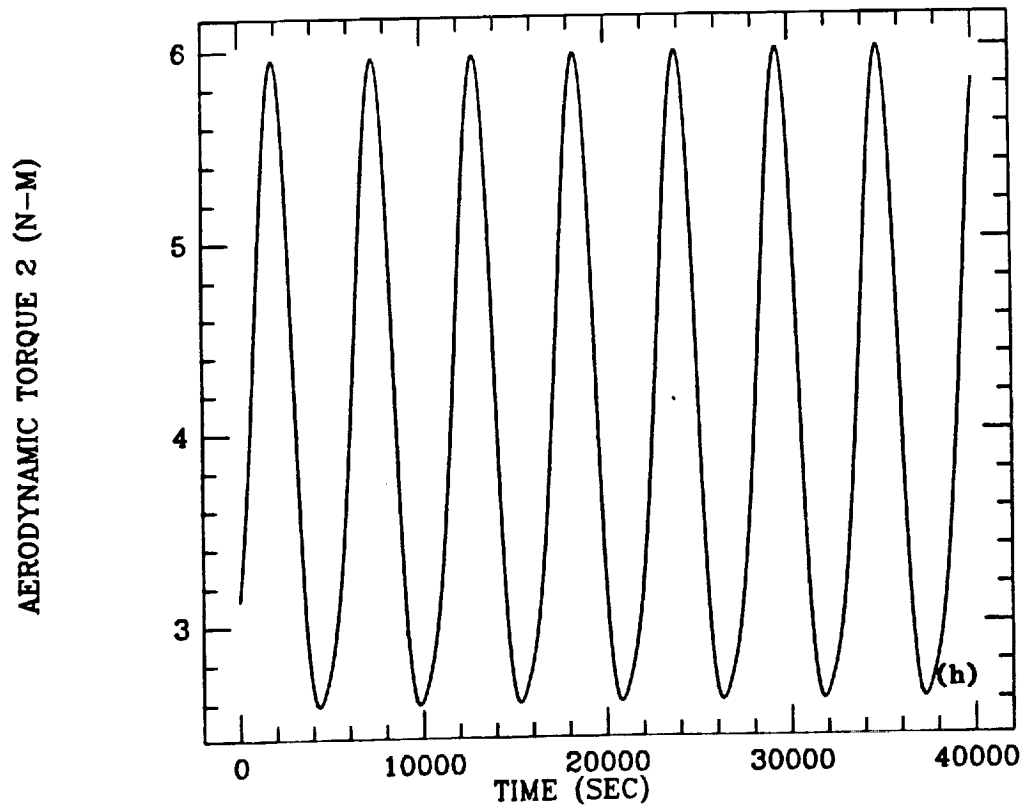
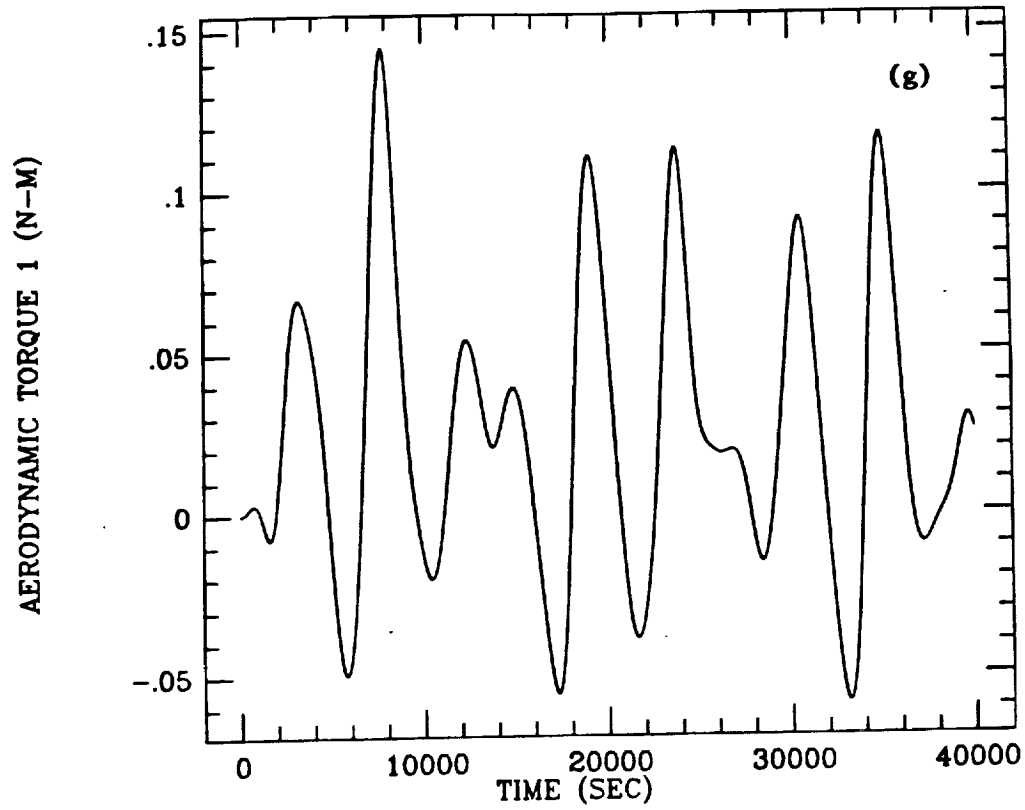
Figures 3a and 3b.



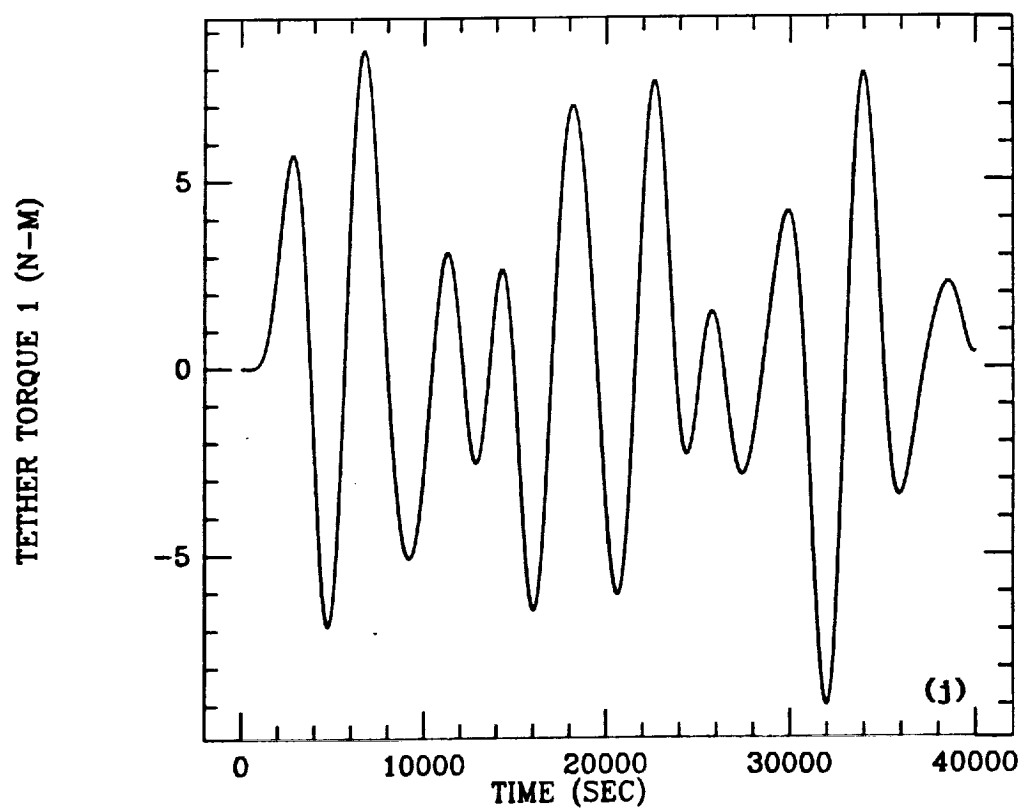
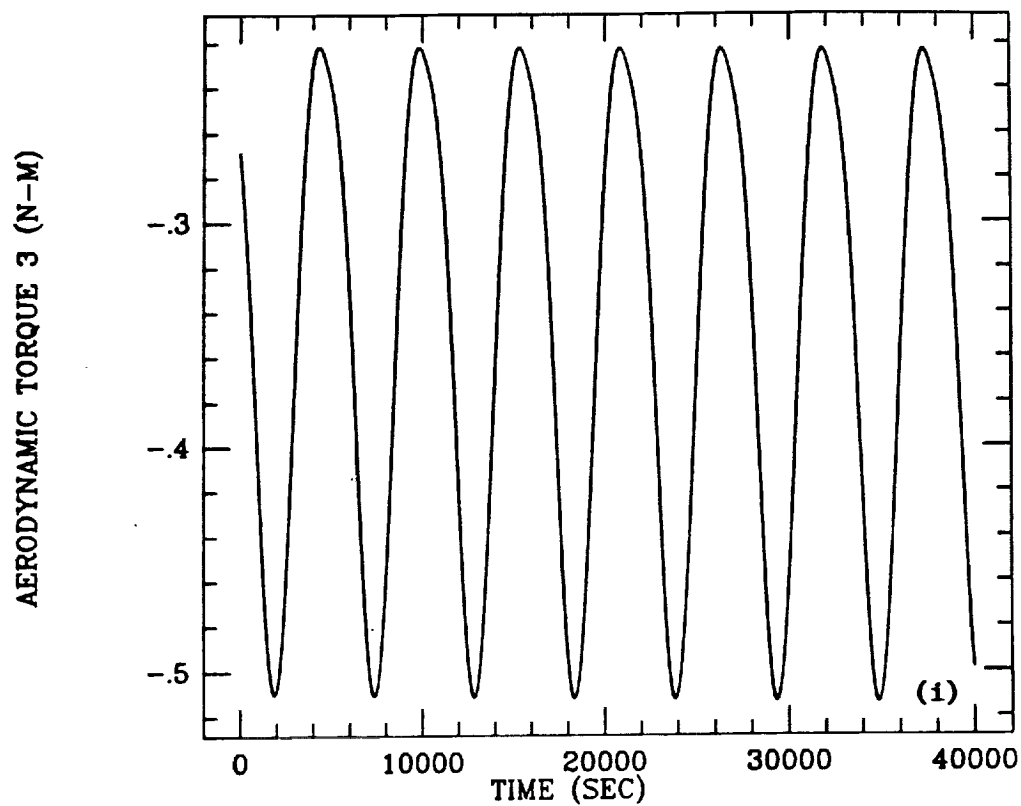
Figures 3c and 3d.



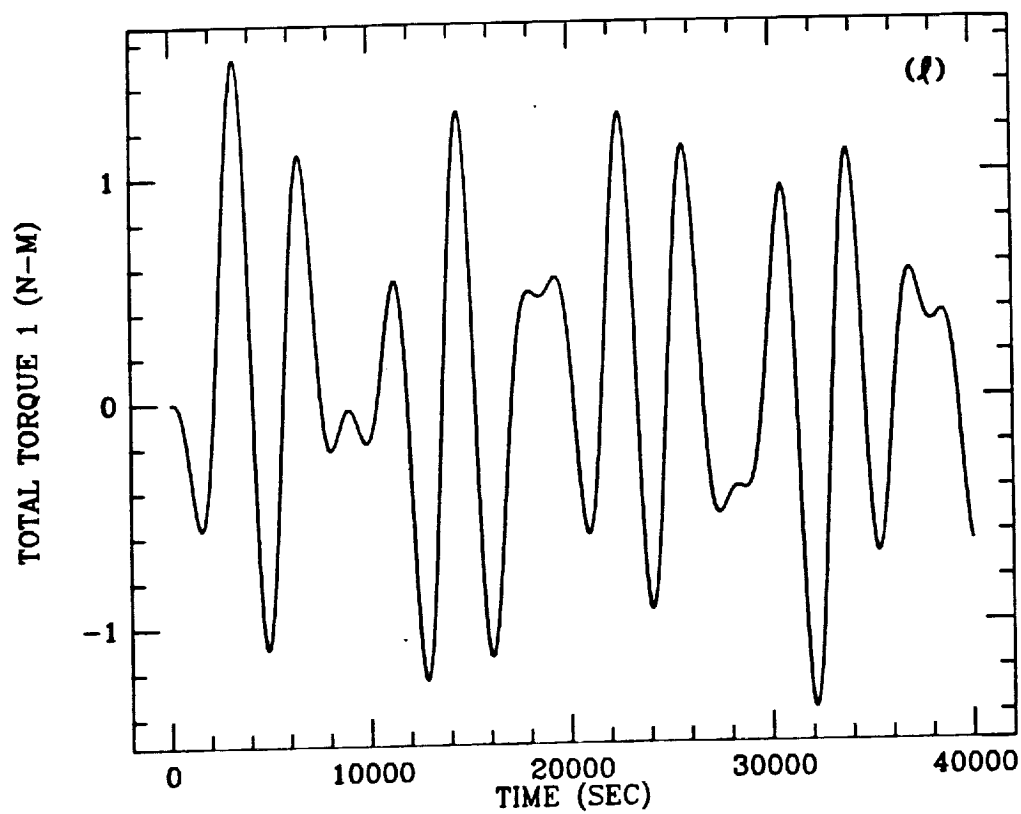
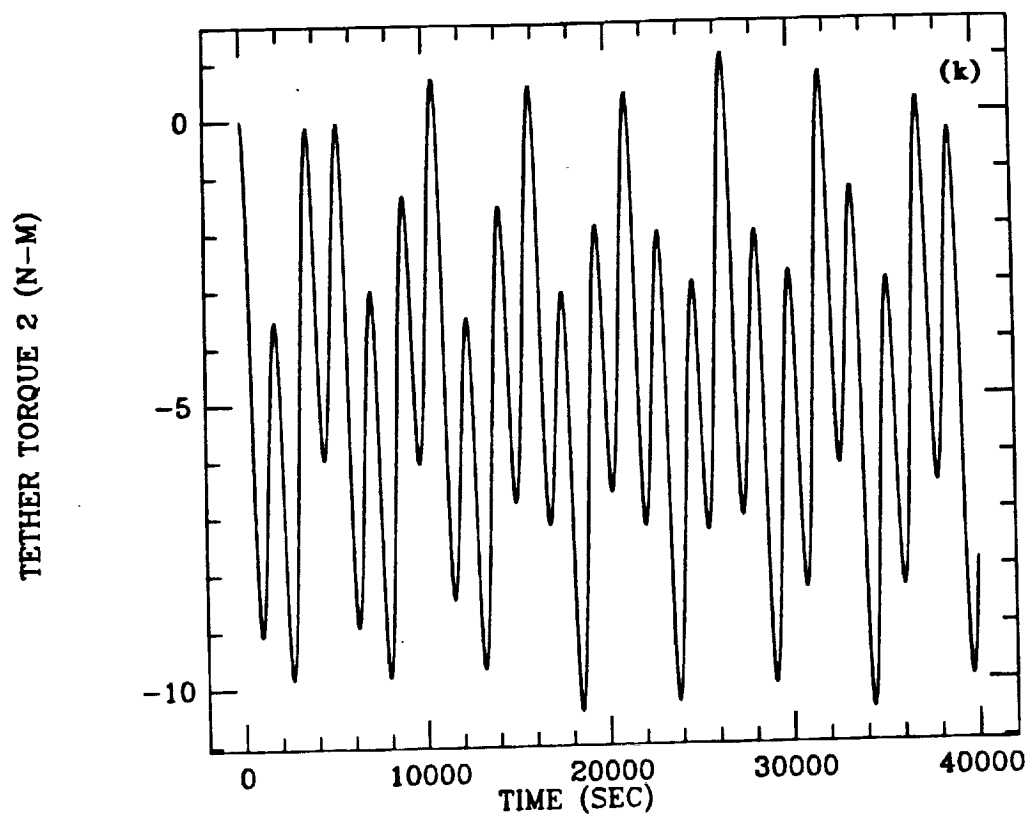
Figures 3e and 3f.



Figures 3g and 3h.

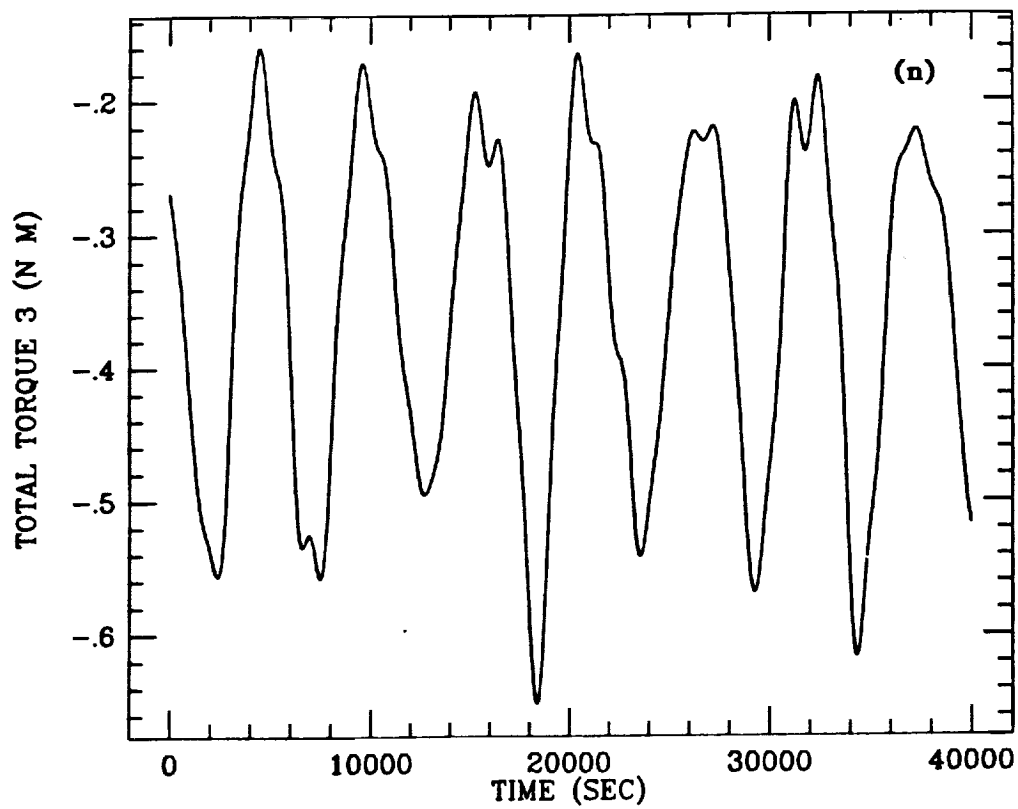
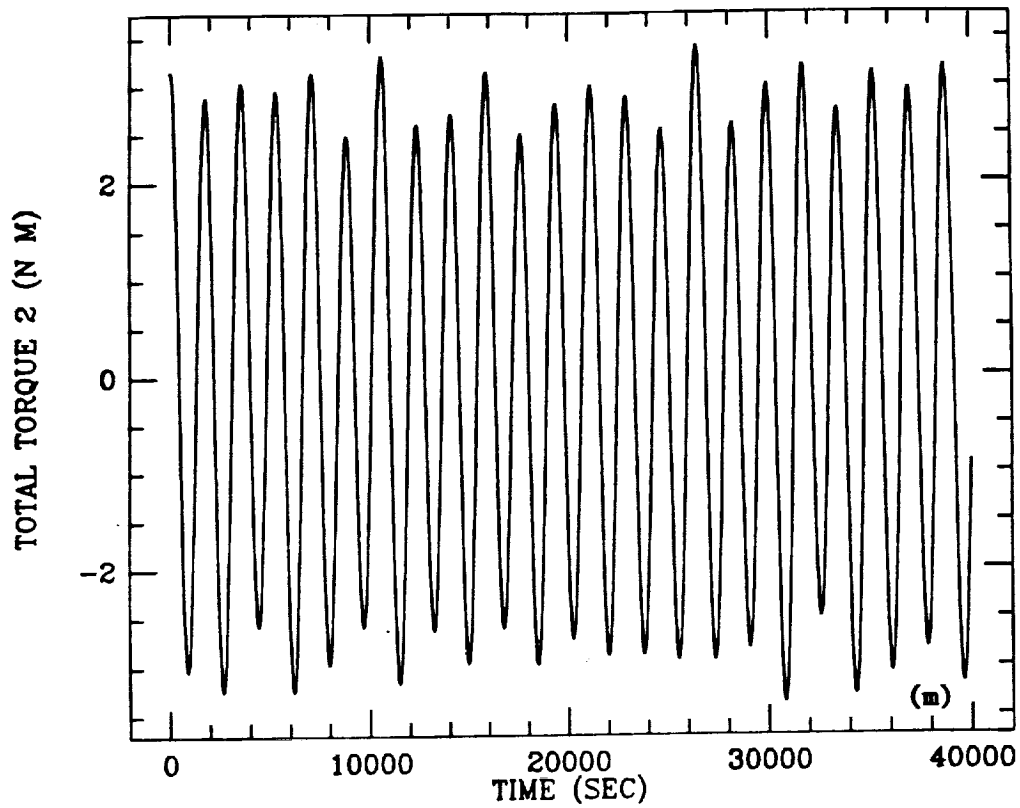


Figures 3i and 3j.



Figures 3k and 3l.

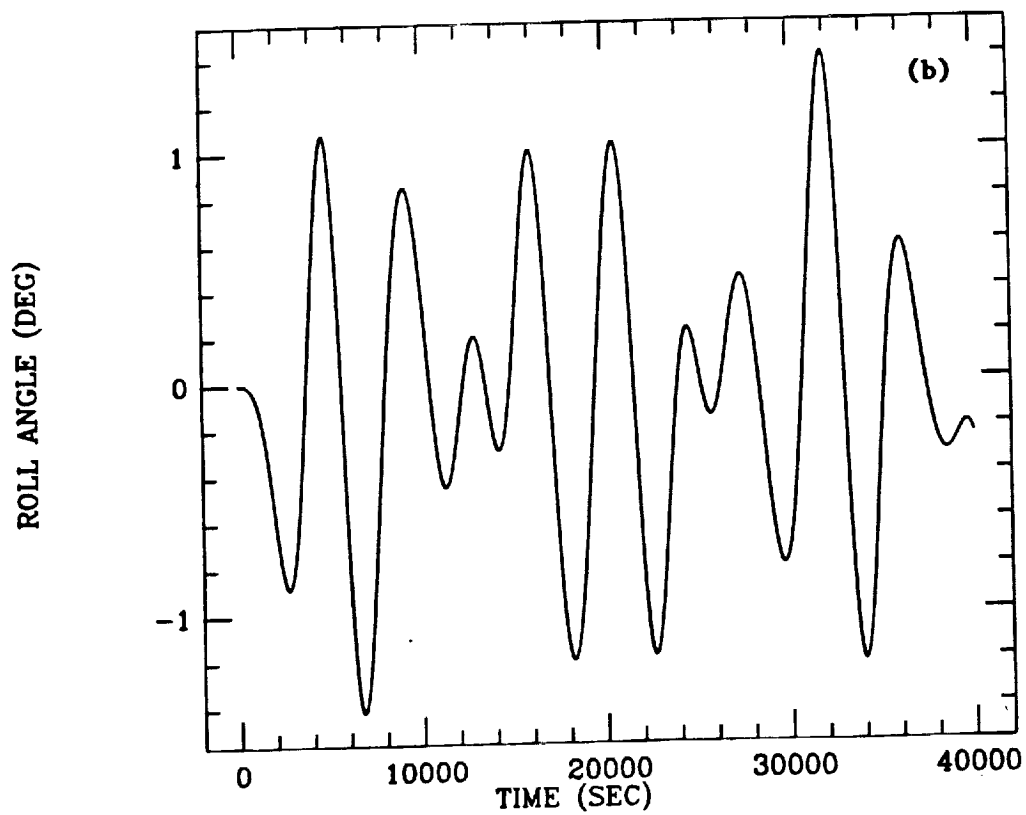
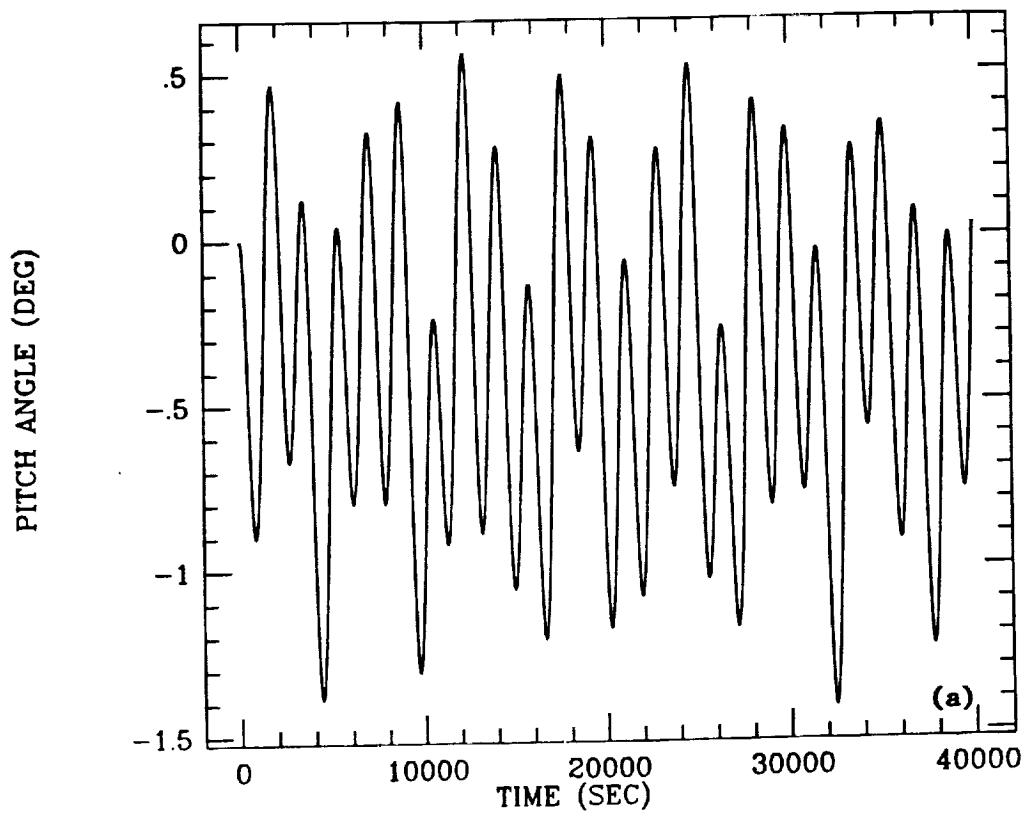




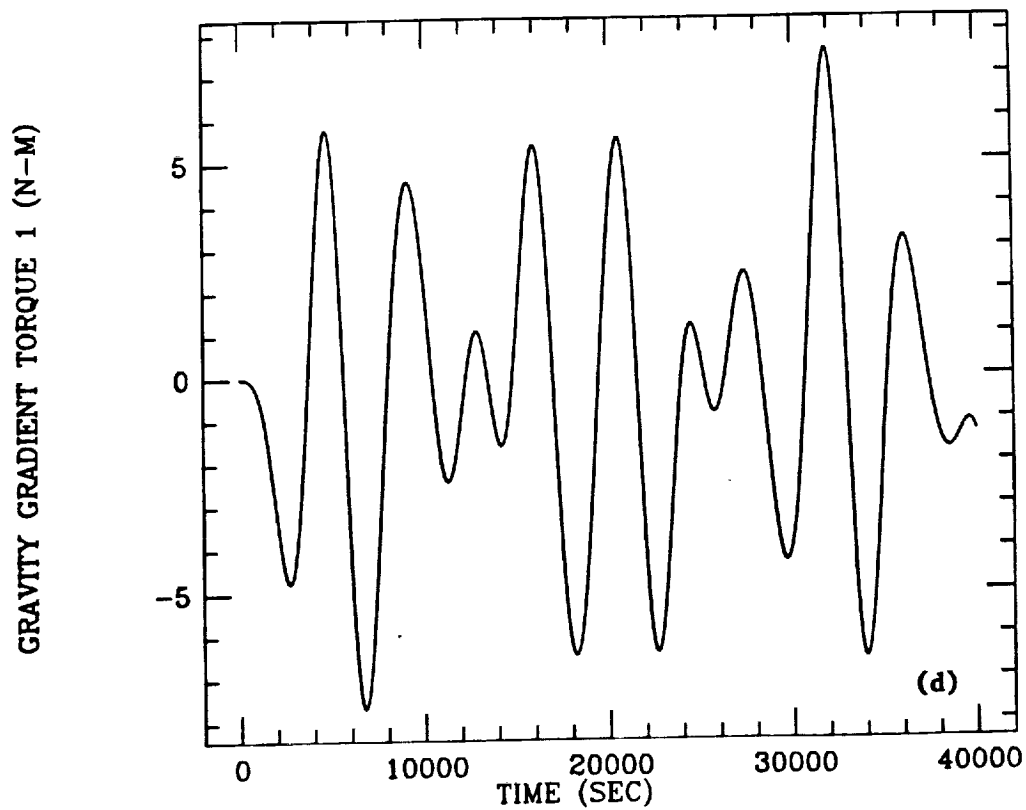
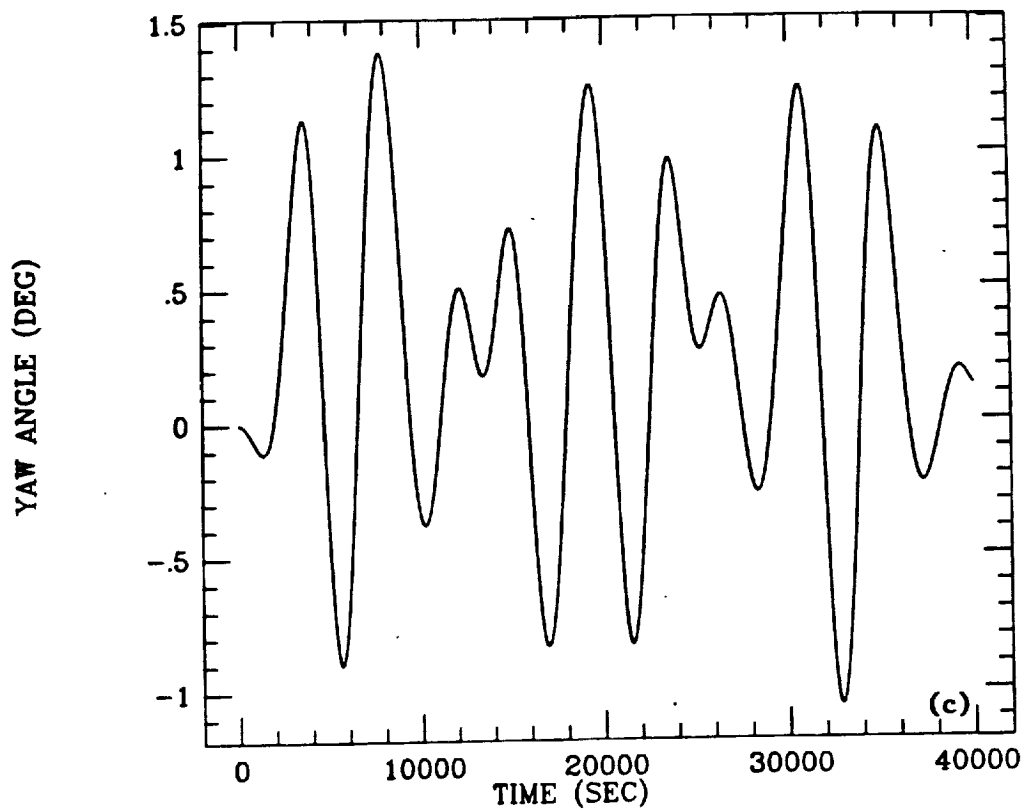
Figures 3m and 3n.

axes in spite of the external perturbations. This conclusion is certainly true for small attitude angles. Further analysis is necessary to evaluate the dynamic response for "non-linear" initial conditions.

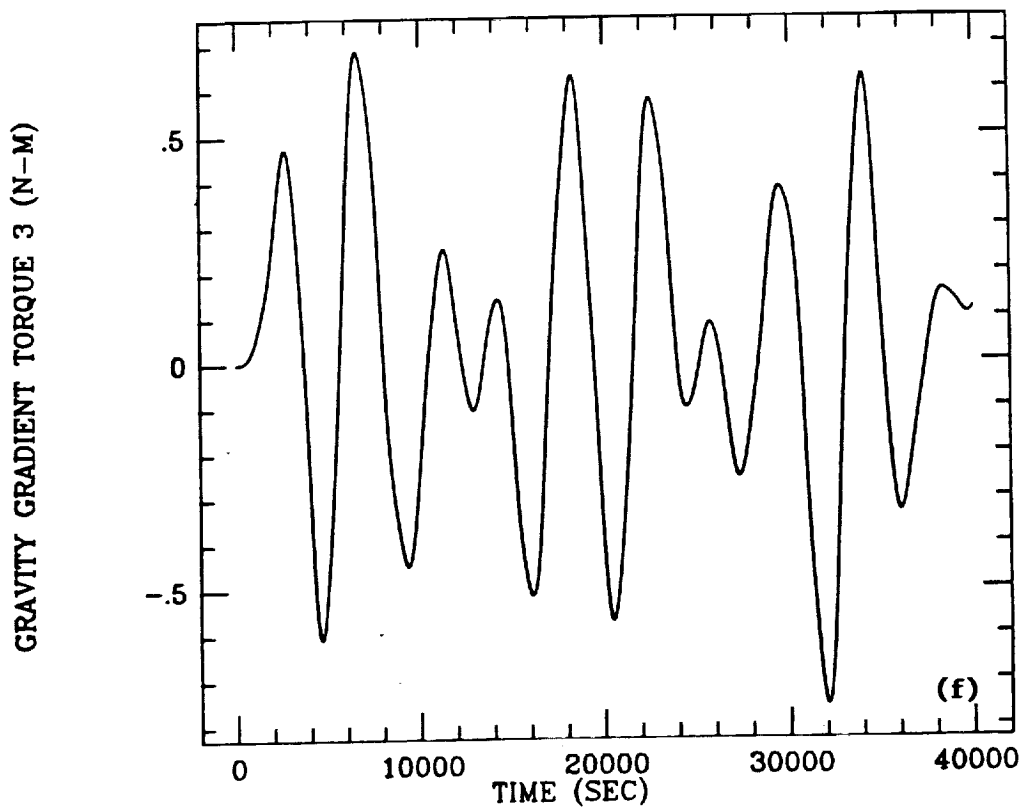
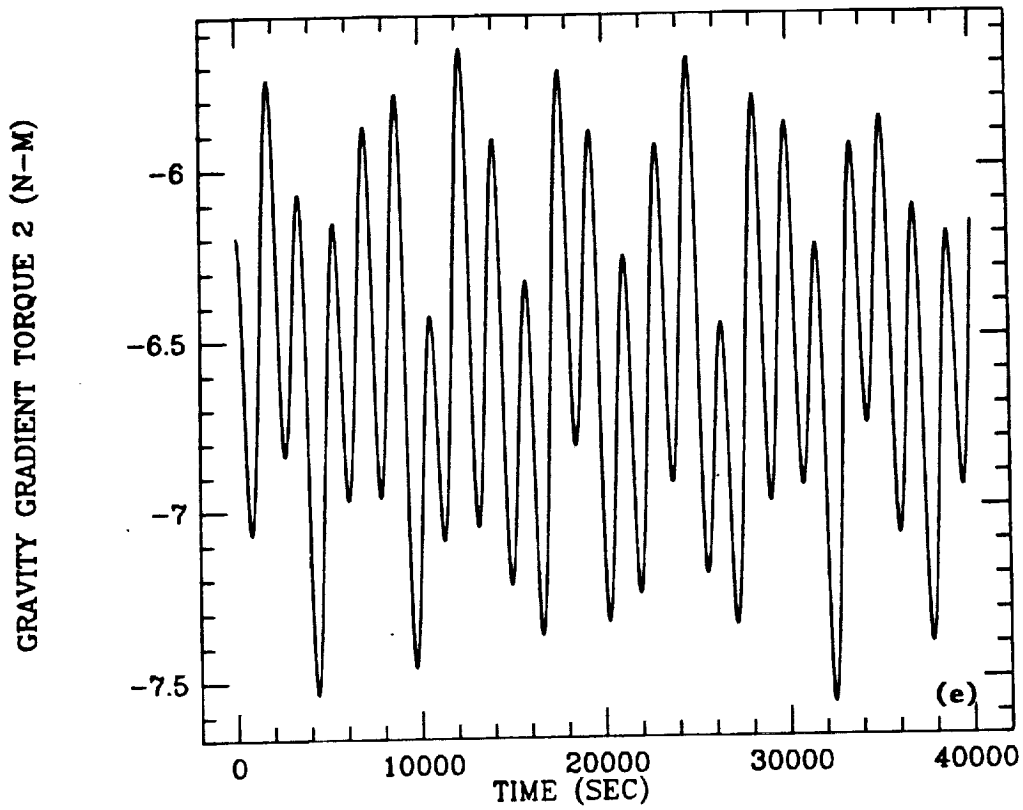
For the sake of completeness the accelerations acting at the SS CG are shown in Figures 4(o), 4(p), and 4(q). Since the tether shifts the system CG, the level of the acceleration component along axis-3 is increased with respect to the baseline level. Notice that these acceleration components are measured at the Station CG and hence the rotational acceleration terms (e.g. centrifugal) are very close to zero because the Station center of rotation almost coincides with the SS CG. The modulation of the acceleration components along axes 1 and 2 is originated by the vertical gravity gradient component projected onto the above mentioned axes.



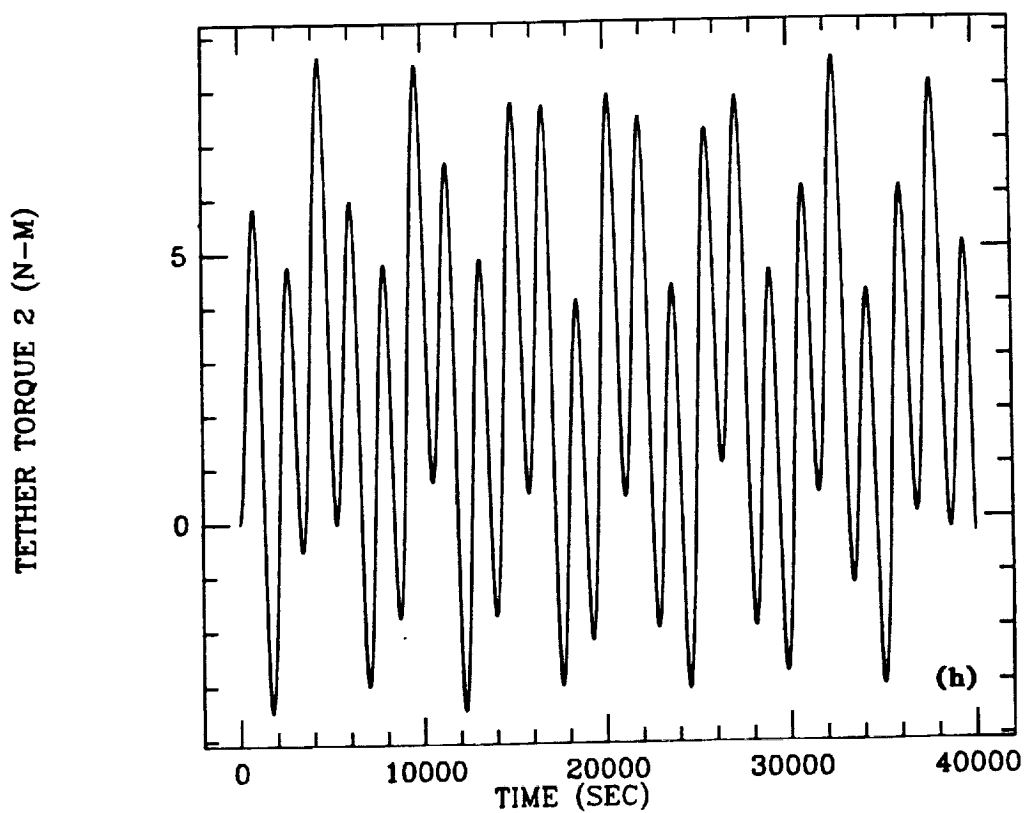
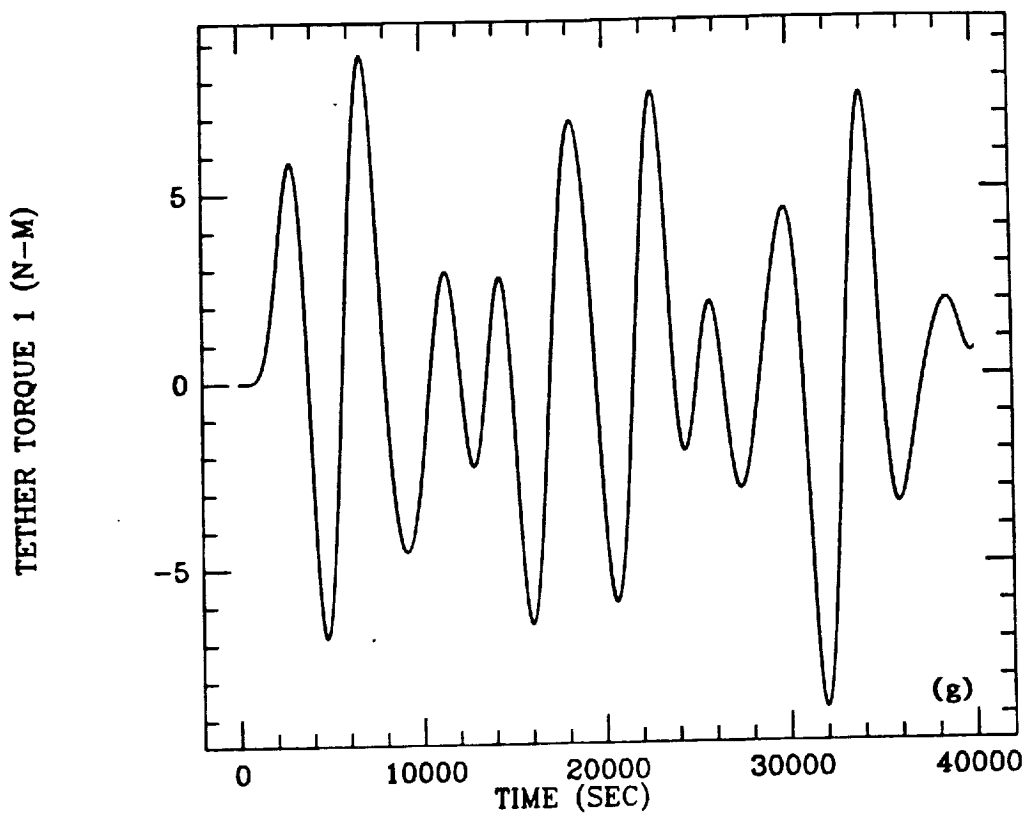
Figures 4a and 4b.



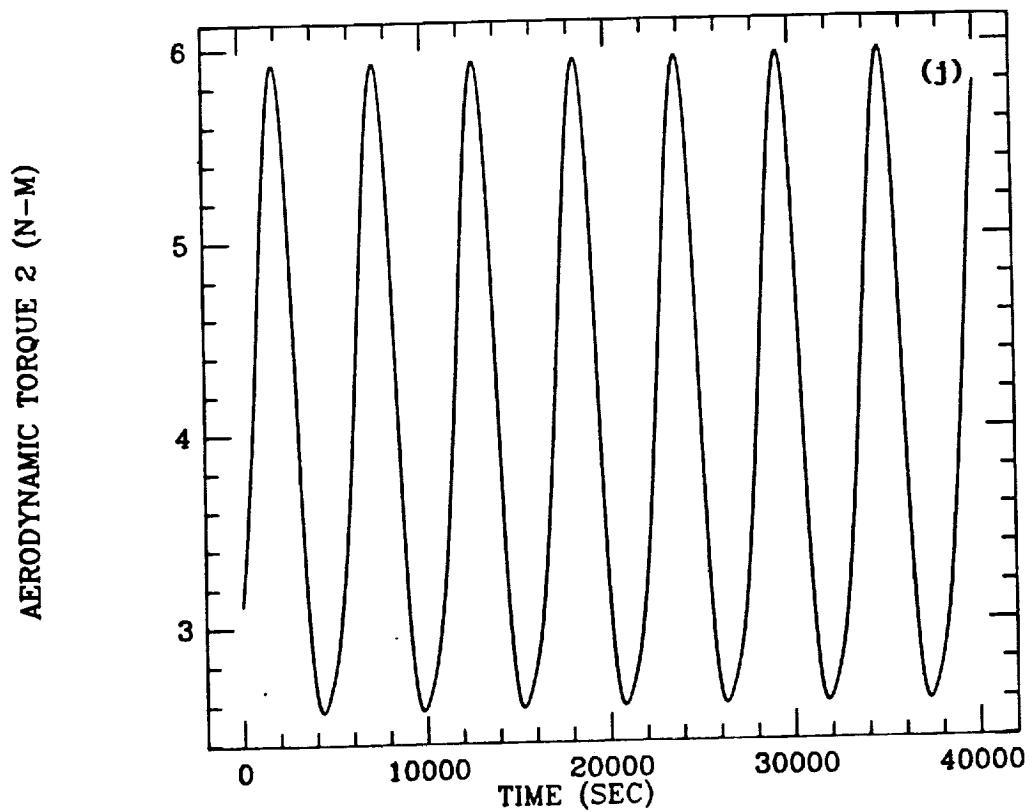
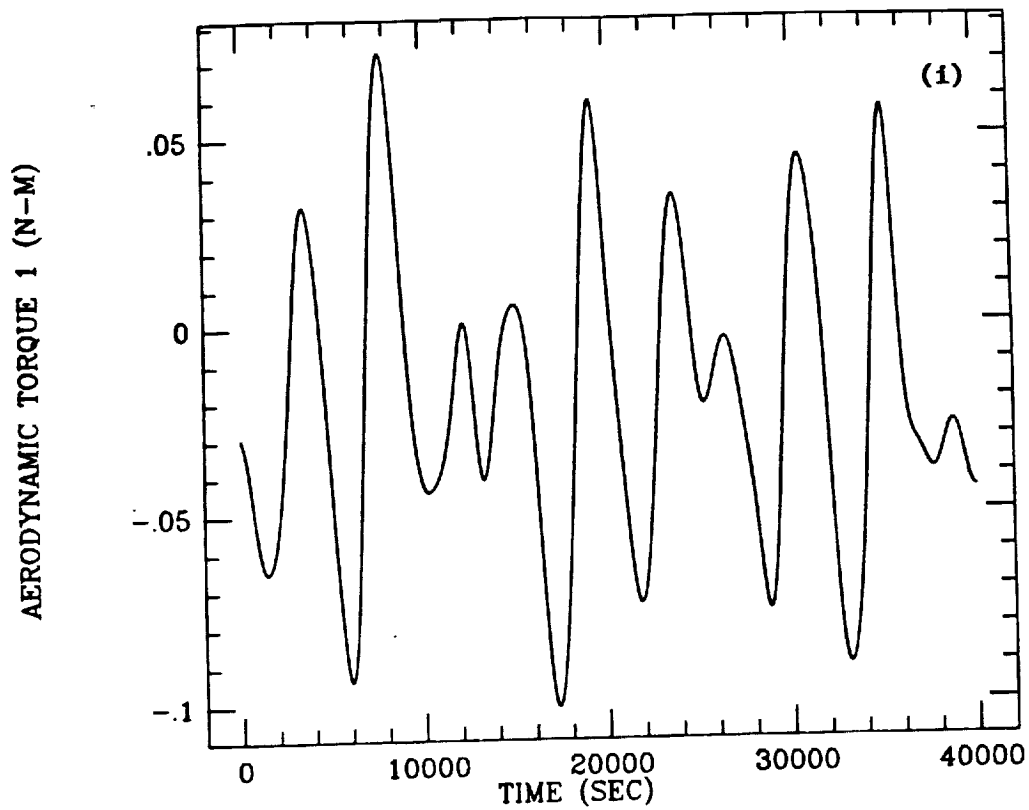
Figures 4c and 4d.



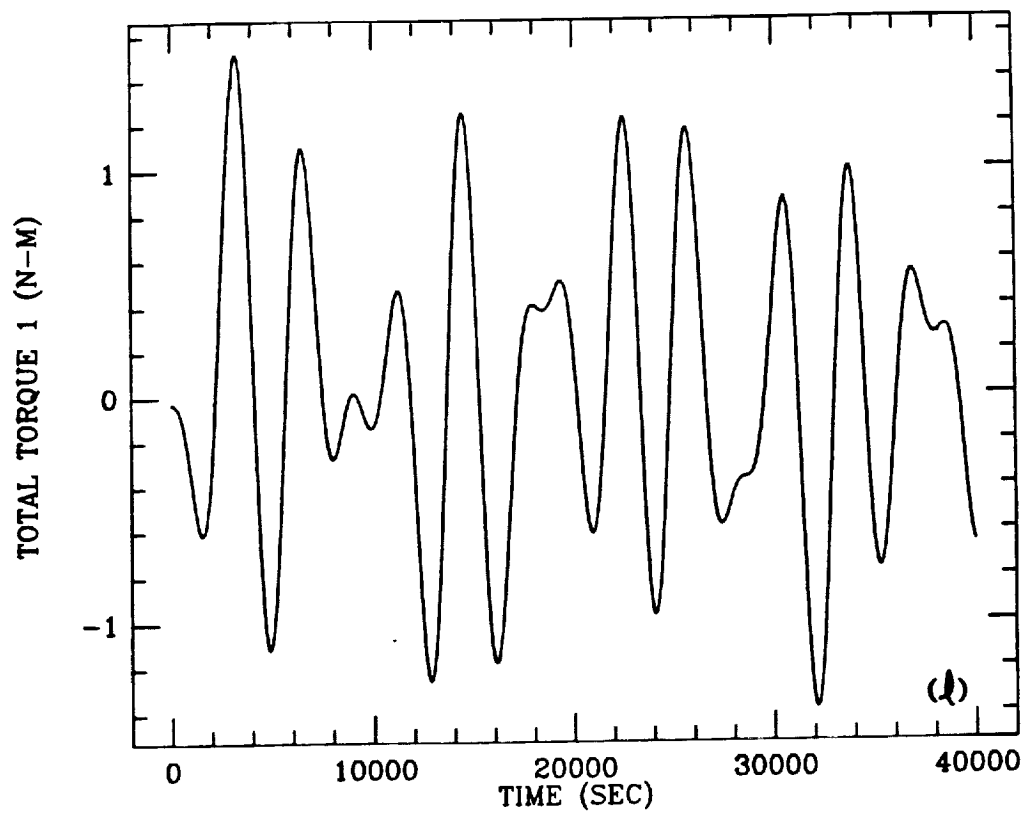
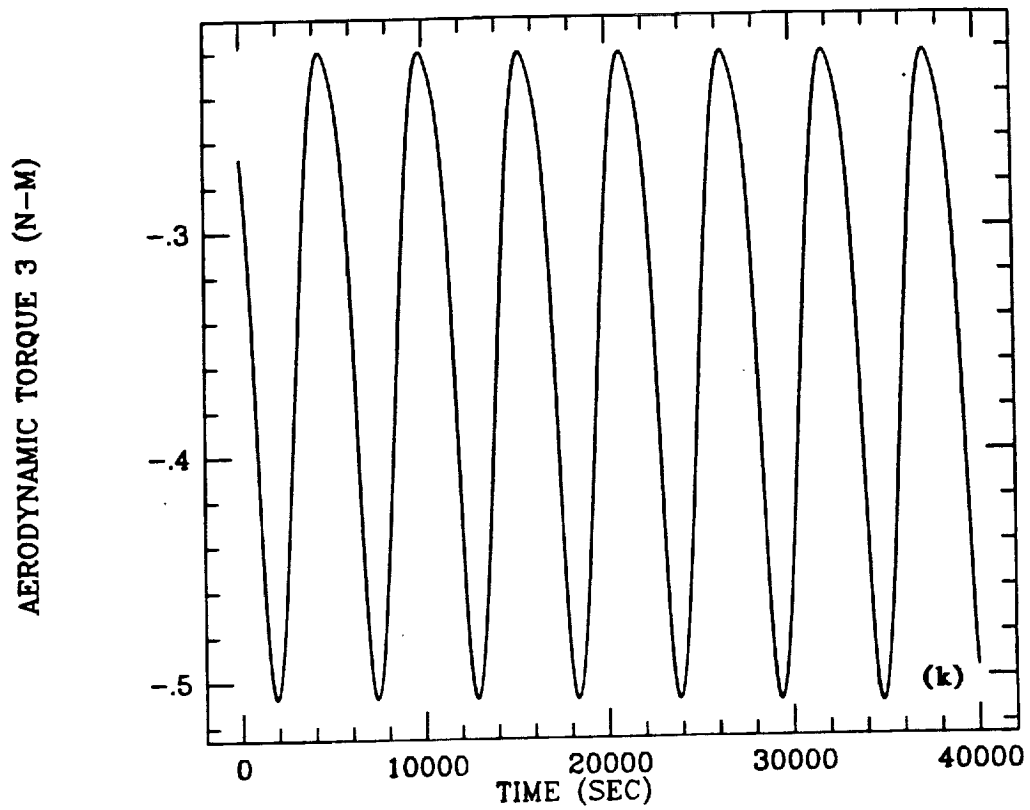
Figures 4e and 4f.



Figures 4g and 4h.

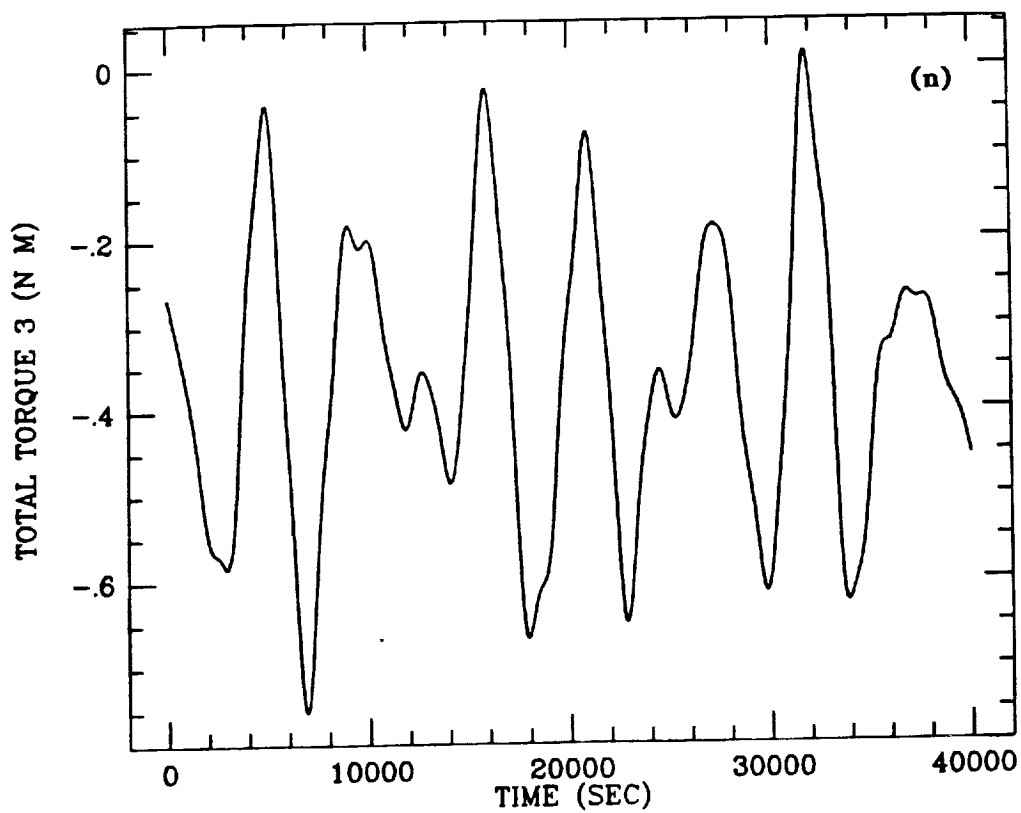
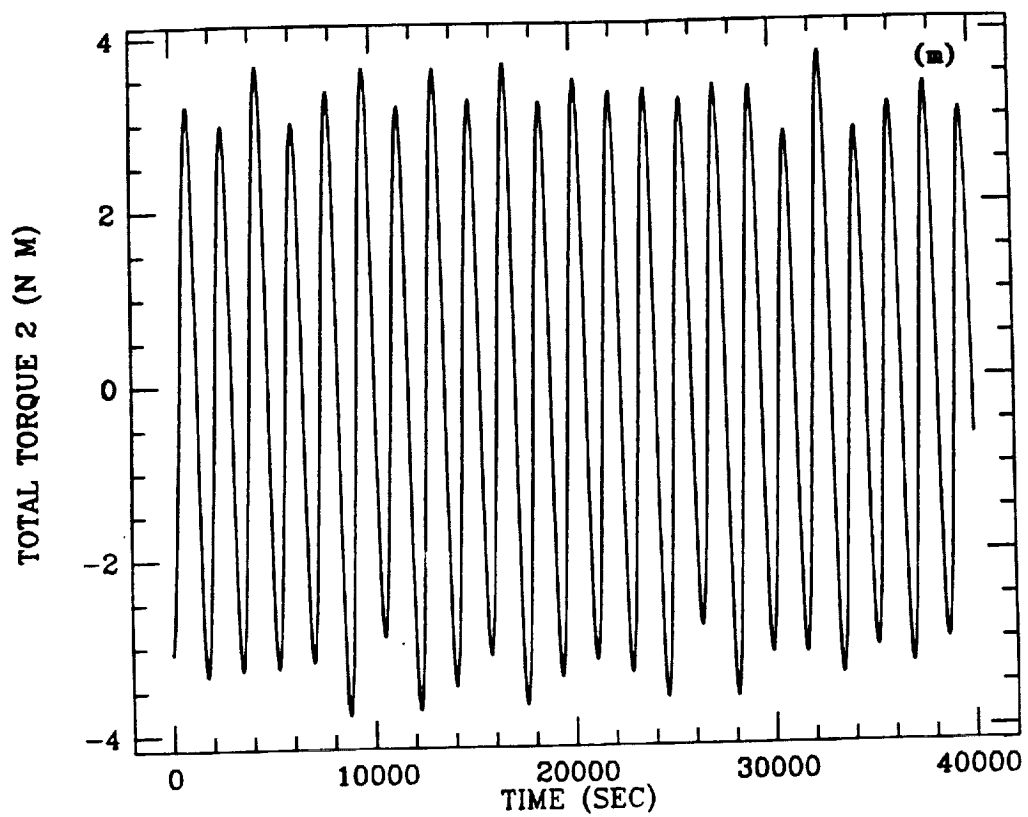


Figures 4i and 4j.

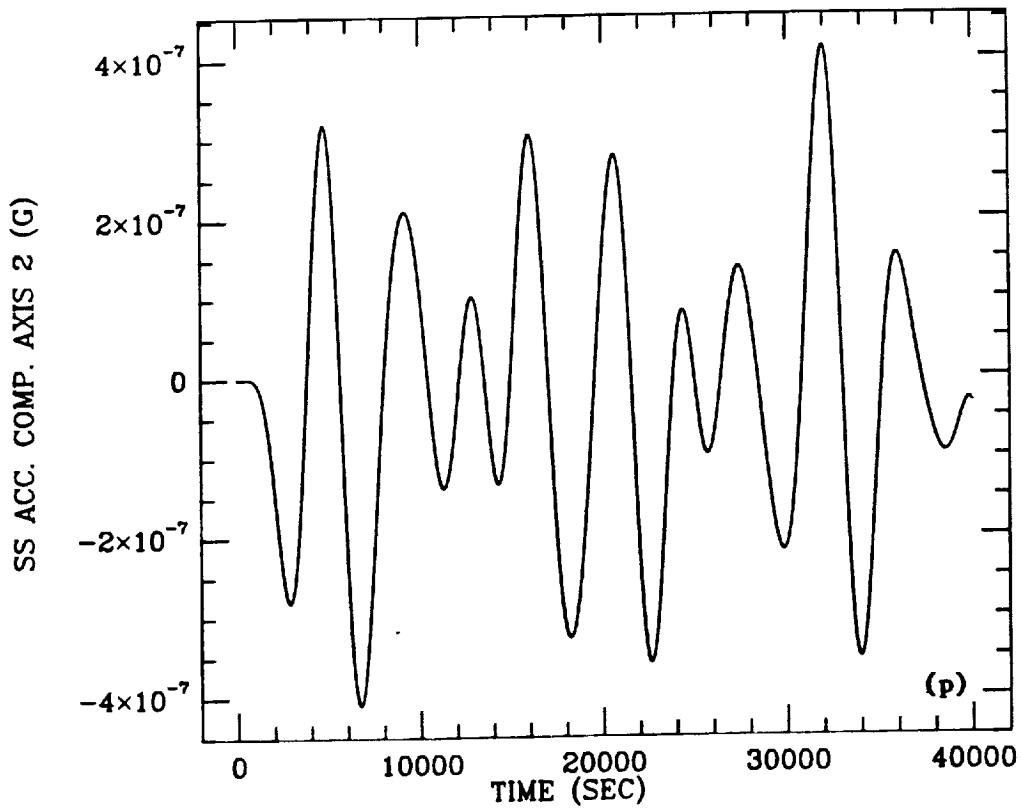
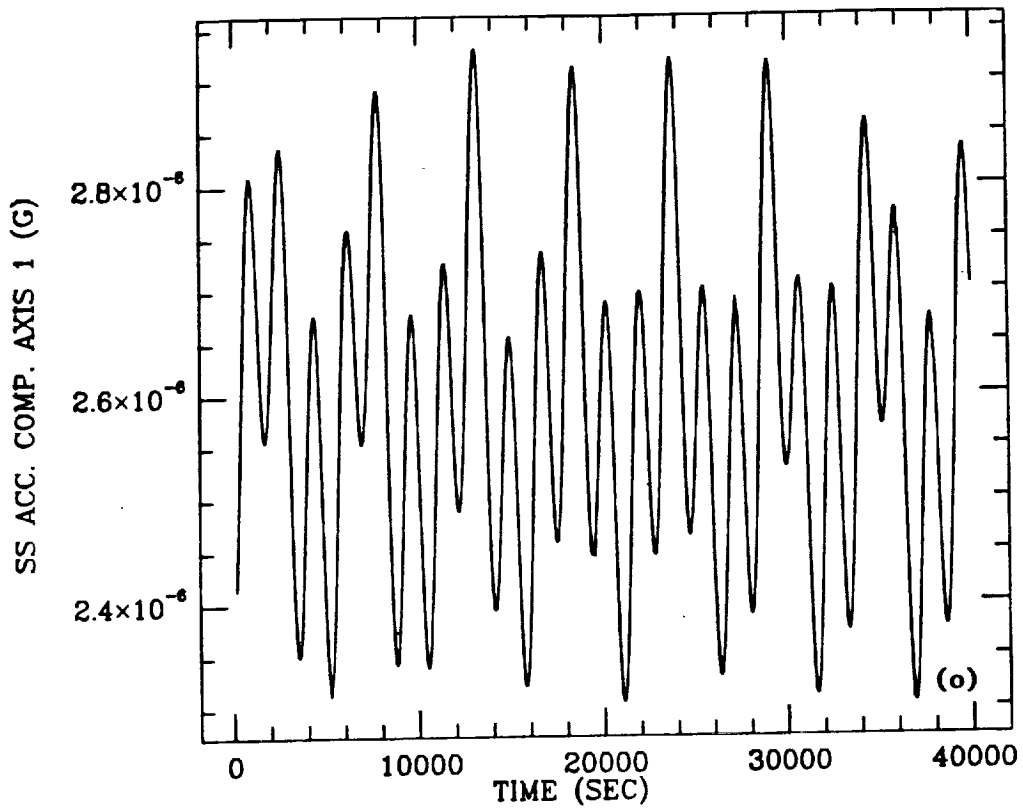


Figures 4k and 4l.





Figures 4m and 4n.



Figures 4o and 4p.

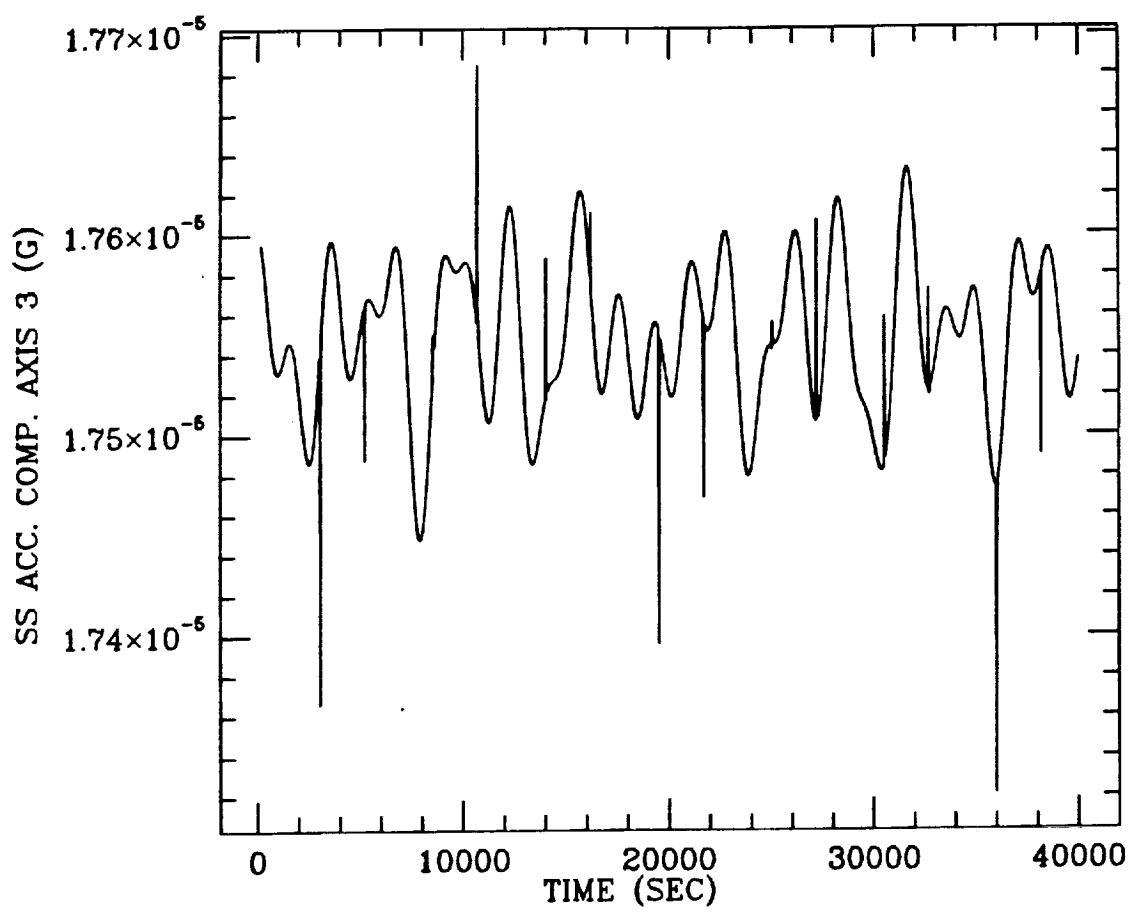


Figure 4q.

## **2.5 Conclusions**

From the analysis conducted thus far we conclude that an appropriately designed tether system is able to control the attitude of the Station around three axes in spite of the environmental perturbations. This conclusion is valid for small amplitudes of the attitude angles. Further analysis is needed in order to assess the performance of the "tether stabilizer" during transient phases and large angular displacements.

## **2.6 References**

- [1] L. DeRyder, P. Trautman and H. Heck, "The Impact of Asymmetric Physical Properties on Large Space Structures," Proceedings of the AIAA Conference SDM for the International Space Station, Williamsburg, VA, April 21-22, 1988.
- [2] F. Bevilacqua, S. Ciardo and A. Loria, "Space Station Gravity Gradient Stabilization by Tethers," Space Tethers for Science in the Space Station Era, Eds. L. Guerriero and I. Bekey, Societa' Italiana di Fisica, Conference Proceedings, Vol. 14, Bologna, Italy, 1988.

## **3.0 PROBLEMS ENCOUNTERED DURING REPORTING PERIOD**

None.

## **4.0 ACTIVITY PLANNED FOR NEXT REPORTING PERIOD**

During the next reporting period the analysis of the tether stabilizer's attitude dynamics will be further developed. Specifically we will assess the dynamic response of the Station for large initial angular displacements.